PETROV-GALERKIN APPROXIMATION FOR ADVECTIVE-DIFFUSIVE HEAT TRANSFER IN SATURATED POROUS MEDIA

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***Abstract*— This article studies the heat transport in a flow through a saturated rigid porous medium. The mechanical model is based on the Continuum Theory of Mixtures which considers the fluid and the porous matrix as overlapping continuous constituents of a binary mixture. A Petrov-Galerkin formulation is employed to approximate the resulting system of partial differential equations, overcoming the classical Galerkin method limitation in dealing with advective-dominated flows. The employed method is built in order to remain stable and accurate even for very high advective-dominated flows. Taking advantage of an appropriated upwind strategy, the applied finite element method proved to generate accurate approximations even for very high Péclet regime. Some two-dimensional simulations of the advective-diffusive heat transfer in a flow through a porous flat channel employing lagrangean bilinear and serendipity biquadratic elements have been performed attesting the reliability of the employed Petrov-Galerkin formulation as well as the poor performance of Galerkin one even when mesh refining is considered.**

***Keywords*— Porous media, mixture theory, computational heat transfer, finite elements, Petrov-Galerkin formulation.**

I. INTRODUCTION

Transport phenomena in porous media play an important role in many a field of engineering science, such as geomechanics, petroleum and mining industries, sintering technologies and biomechanics. Besides, interactions among fluids and solids are present in many industrial processes and the fluids may be passed over packed beds of solid material in order to improve processes like heat and mass transfer or chemical reactions. Nowadays an increasing attention is being devoted to transport in porous media motivated by the importance of problems that impact the energy self-sufficiency and the environmental state. Some practical applications like packed-bed heat exchangers, enhanced oil recovery processes, storage of nuclear waste material, contamination of soils by hazardous wastes and pollution movement stimulate the interest attached to these phenomena.

Most of the works dealing with transport in porous media describe quantities such as temperature, pressure, concentration and the velocity components as volumetric averages (Whitaker, 1969); in order to describe the phenomena by employing a classical continuum mechanics approach. These models substitute the balance of linear momentum by Darcy's law with the addition of empirically determined terms – Brinkmann and Forchheimer extensions – to account for inertia and viscous effects and to satisfy the no-slip condition (Vafai and Tien, 1981). Nield (2000) analyzed viscous dissipation and nonlinear drag for Darcy, Brinkmann and Forchheimer models for incompressible fluid flows through porous media. The so-called volume averaging technique has already allowed the analysis of complex problems. Examples are the multiphase transport process with phase change in unsaturated porous media (Vafai and Whitaker, 1986), the forced convection considering heat sources and a partially porous channel (Hadim, 1994) or axial and radial dispersion (Adani *et al.*, 1995), the mixed convection (Aldouss *et al.*, 1996; Chang and Chang, 1996; Chen *et al.*, 1996) as well as variable porosity effects (Vafai, 1984). Thermally developing forced convection in a porous medium was studied by Nield *et al.* (2003) employing a modified Graetz method with Brinkmann model, for parallel plate channel and circular tube, both with walls at constant heat flux. This work was subsequently extended considering walls at constant temperature (Nield *et al.*, 2004a), and both boundary conditions were considered by Nield *et al.* (2004b). The entropy generation – considering viscous dissipation effects (Brinkmann extension) – was analyzed by Mahmud and Fraser (2005) who obtained analytical expressions for velocity, temperature and Nusselt number. Hooman and Ejlali (2007) analyzed thermally developing forced convection in a porous matrix employing both First and Second Laws of Thermodynamics and including Brinkmann (viscous dissipation) effects, using the perturbation solution of Hooman and Ranjbar-Kani (2004), in order to compute

the entropy generation. Brinkmann model was carefully analyzed by Al-Hadrami *et al.* (2003), who derived a mathematical theory presenting the correct asymptotic behavior for Darcy limit.

Most works assume the thermal equilibrium hypothesis so that the balance of energy may be expressed as a single equation, requiring the determination of an effective thermal conductivity. The thermal nonequilibrium hypothesis, although seldom employed, has already been considered by several authors, such as Vafai and Sozen (1990), presenting a detailed analysis of non isothermal flows through a packed bed, Sozen and Vafai (1990), studying a condensing gas flow through a packed bed and Nield (1998), presenting an analytic solution for the forced convection in a saturated porous medium limited by a parallel-plane channel and concluding that local thermal nonequilibrium reduces Nusselt number. Nield and Kuznetsov (1999) presented an analytical solution for the forced convection in a saturated porous medium limited by a parallel-plane channel – coupling with conduction in the plane slabs bounding the channel, and discussing boundary conditions. Nield *et al.* (2002) studied the effect of local thermal non-equilibrium on the thermal development of forced convection in a saturated porous medium limited by a parallel-plane channel – by employing the classical Graetz methodology combined with the Brinkman model.

In this article, the heat transfer in a saturated flow through a rigid porous medium has been studied by employing a local model based on a *Continuum Theory of Mixtures* (Atkin and Craine, 1976; Bedford and Drumheller, 1983; Bowen, 1967). This model treats the mixture as a superposition of continuous constituents – each of them occupying its whole volume. Herein a binary mixture is considered, whose constituents stand for the fluid and the porous medium; the fluid constituent is assumed Newtonian and incompressible, while the solid constituent, representing the porous matrix, is supposed rigid, homogeneous, isotropic and at rest (Martins-Costa *et al.*, 1992; Costa-Mattos *et al.*, 1995).

An approach, distinct from the one mentioned in the previous item, is used in this work: the Continuum Theory of Mixtures – supported by a local theory with thermodynamic consistency – which generalizes the classical continuum mechanics (Truesdell, 1957; Truesdell and Toupin, 1960; Gurtin, 1981). It is employed to model flows through porous media in which the fluid and the porous matrix are treated as superimposed continuous constituents of a binary mixture. The mixture theory leads to an apparent thermomechanical independence allowing the existence of n distinct velocity fields and n distinct temperature fields (if the flow is not assumed isothermal), simultaneously, at each spatial point, whenever an n-constituents mixture is considered. In order to provide dynamical and thermal interactions, additional terms, absent in a Continuum Mechanics description – playing the role of momentum and energy sources – are required to account for the thermomechanical coupling among the constituents in the balance equations. Considering the solid constituent, which represents the porous matrix, rigid and at rest, it suffices to solve mass and momentum conservation equations for the fluid constituent of the mixture, while the energy equation must be solved for both fluid and solid constituents. These equations, combined with constitutive assumptions satisfying the material objectivity and the Second Law of Thermodynamics, describe the heat convection in a porous medium (Martins-Costa *et al.*, 1992; 1993; Martins-Costa and Saldanha da Gama, 1994a; 1994b; Costa-Mattos *et al.*, 1995).

Finite element approximations of incompressible flows suffer from two major difficulties (Johnson, 1987; Pironneau, 1980). First, finite elements need to compatibilize velocity and pressure subspaces satisfying the Babŭska-Brezzi mathematical condition (Ciarlet, 1978). The second one, the instability inherent to central discretization schemes, either by Galerkin formulation or by central difference stencil, to approximate high advective dominated flows (Brooks and Hughes, 1982; Patankar, 1980).

Simple strategies have been proposed that may overcome most of the limitations found in the Galerkin method when applied to fluid problems (Brooks and Hughes, 1982; Hughes *et al.*, 1986; Tezduyar *et al.*, 1990; Sampaio, 1991; Franca *et al.*, 1992; Franca and Frey, 1992). In this article, in order to perform the numerical simulations, a Petrov-Galerkin formulation for advective-diffusive heat transport in porous media is employed. The formulation was built in adding mesh-dependent terms to the usual Galerkin formulation, which are functions of the Euler-Lagrange equations evaluated element wise. The perturbation terms are designed to enhance stability of the original Galerkin formulation. Taking advantage of the design of the stability parameter introduced in Franca *et al.* (1992) for the scalar context of advective-diffusive equation, it remains stable even for very high advective flows.

II. MECHANICAL MODELING

The Continuum Theory of Mixtures, based on principles postulated by Fick and Stefan (Atkin and Craine, 1976; Bedford and Drumheller, 1983; Bowen, 1967), is a convenient framework for modeling multicomponent systems whose basic assumption is to regard the mixture as a superposition of $n \geq 2$ continuous constituents (each of them representing a material). These constituents are endowed with independent kinematics, in order that for any time instant t and position $\mathbf{x}$ the mixture is occupied by all the n distinct constituents. In this section, the kinematics of motion and the axioms of balance of mass, momentum and energy for a mixture of n chemically non-reacting bodies B_α, α=1, n are considered following the definitions stated by Atkin and Craine (1976) and Bowen (1967). Each body B_α is a set

with the structure prescribed by Noll (Bowen, 1967) and a particle in this body is denoted by P_α.

A configuration of the body B_α is a homeomorphism χ of B_α onto a subset of the Euclidean space ε. Its motion is a one-parameter family of configurations χ_t where $t \in [0,\infty)$ is the time (Fig. 1). The position of the particle B_α at the time t is given by

$$\mathbf{x} = \chi_t(P_\alpha) = \chi_\alpha(P_\alpha, t)\,. \tag{1}$$

A reference configuration for the body B_α is a fixed configuration ζ_α, which is denoted by

$$\chi_\alpha = \zeta_\alpha(P_\alpha)\,. \tag{2}$$

It follows from Eqs. (1) and (2) that,

$$\mathbf{x} = \chi_\alpha\left(\zeta_\alpha^{-1}(\mathbf{X}_\alpha, t)\right) = \chi_{\zeta\alpha}(\mathbf{X}_\alpha, t)\,. \tag{3}$$

This equation yields the motion of the point P_α in terms of its position $\mathbf{X}_\alpha$ in the reference configuration ζ_α. The function $\chi_{\zeta\alpha}$ is called the deformation function of the body B_α. From now on both $\chi_{\zeta\alpha}$ and its inverse $\chi_{\zeta\alpha}^{-1}$ are assumed C^2 functions of their arguments.

The region occupied by B_α in the Euclidean space ε is a compact set with piecewise smooth boundaries defined by

$$\chi_\alpha(B_\alpha, t) = \left\{\chi_\alpha(P_\alpha, t) \middle| P_\alpha \in B_\alpha\right\}. \tag{4}$$

A position in ε may be occupied simultaneously by material points from each body as in the Continuum Theories of Mixtures (Bowen, 1967). The mixture at a given time t, denoted by B_t, is defined by

$$B_t = \bigcup_{\alpha=1}^{n} \left\{P_\alpha \middle| \chi_\alpha\left(P_\alpha, t\right) \in \mathcal{V}\left(B_t\right)\right\} \tag{5}$$

where $\mathcal{V}(B_t)$ is a region defined by

$$\mathcal{V}\left(B_t\right) = \bigcap_{\alpha=1}^{n} \left\{\chi_\alpha\left(B_\alpha, t\right)\right\} \tag{6}$$

As a result of the definition of $\mathcal{V}(B_t)$ and the requirement that each function χ_α be a homeomorphism, it follows that each point $\mathbf{x} \in \mathcal{V}\left(B_t\right)$ is occupied by exactly n particles, one from each body. The part of B_α that is in the mixture B_t at the time t is clearly given by $B_\alpha \cap B_t$.

A mass density ρ_α is associated with each constituent, representing the average density of the α-constituent taken over a small volume of the mixture. A velocity field $\mathbf{u}_\alpha$ is also associated to each constituent. Its Lagrangean and Eulerian description being given, respectively, by

$$\mathbf{u}_\alpha = \frac{\partial}{\partial t}\chi_\alpha\left(\mathbf{X}_\alpha, t\right) \quad \mathbf{u}_\alpha = \mathbf{u}_\alpha\left(\mathbf{x}, t\right) \quad \alpha = 1, n\,. \tag{7}$$

The above stated kinematic definitions allow the existence of n different velocities at a single spatial point for any time instant. Figure 1 clarifies the mixture movement in which n reference configurations correspond to a single current configuration.

Let Ω be an arbitrary region fixed in space of volume V bounded by a surface $\partial\Omega$ of area A on which the conservation laws are postulated for each constituent of the mixture, taking into account the action it suffers from the remaining constituents. In what follows all the equations are postulated at the current time t and all quantities are functions of $\mathbf{x}$ and t.

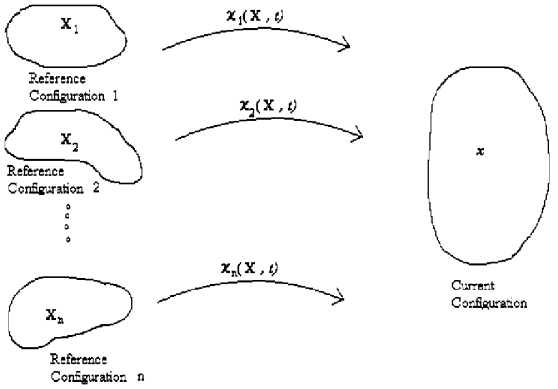

Figure 1. Mixture theory.

In the absence of chemical reactions the amount of any given constituent remains unchanged so that the mass of each constituent is preserved. Therefore, assuming that $\mathbf{n}$ is the unit outward normal to $\partial\Omega$ it can be mathematically stated, for $\alpha = 1, n$

$$\frac{d}{dt}\int_\Omega \rho_\alpha dV + \int_{\partial\Omega} \rho_\alpha \mathbf{u}_\alpha \cdot \mathbf{n} dA = 0 \tag{8}$$

Applying the divergence theorem to Eq. (8) and assuming all functions sufficiently regular, since Ω is an arbitrary fixed region within the mixture, the local form of Eq. (8), the so called continuity equation for each constituent ($\alpha = 1, n$), is obtained

$$\frac{\partial \rho_\alpha}{\partial t} + \left(\nabla \rho_\alpha\right)\mathbf{u}_\alpha + \rho_\alpha \nabla \cdot \mathbf{u}_\alpha = 0 \tag{9}$$

Once Eq. (9) is valid for all α-constituents, the mass of the mixture as a whole is automatically preserved.

The conservation of linear momentum is postulated in an analogous way to the one employed in classical continuum mechanics, by applying the first axiom of Euler (Truesdell, 1957) to each constituent of the mixture. Besides the body force per unit mass acting on each constituent, $\mathbf{f}_\alpha$, the surface force effect as well as the effect of the remaining constituents of the mixture over α must be taken into account. In order to consider these two effects the partial stress vector $\mathbf{t}(\mathbf{x}, t; \mathbf{n})$ – defined on $\partial\Omega$ and measured per unit area of $\partial\Omega$, playing a role in the mixture theory corresponding to the stress vector in continuum mechanics – and an interaction force $\mathbf{m}_\alpha$ applied on the α-constituent by the remaining constituents of the mixture are introduced. This latter effect represents the momentum transfer due to interaction effects such as the allowed relative motion of the constituents. It may be regarded as the diffusive force acting on the α-constituent by the remaining ones. (In the particular case of a fluid flowing through a porous medium, the interaction force acting on the porous matrix may be physically viewed as analogous to the fluid drag on the matrix.) The partial stress vector integrated over a surface $\partial\Omega$ stands for the contact

force on the α-constituent and, since the theorem of Cauchy states that the traction vector is linear on the outward normal $\mathbf{n}$, $\mathbf{t}(\mathbf{x}, t; \mathbf{n}) = \boldsymbol{\sigma}(\mathbf{x}, t)\mathbf{n}$ – in which $\boldsymbol{\sigma}$ represents Cauchy stress tensor, the partial stress tensor $\boldsymbol{\sigma}_a$ may be defined in a similar way, $\mathbf{t}_a(\mathbf{x}, t; \mathbf{n}) = \boldsymbol{\sigma}_a(\mathbf{x}, t)\mathbf{n}$, so that the conservation of linear momentum for each constituent ($\alpha = 1, n$) may be postulated as

$$\frac{d}{dt}\int_\Omega \rho_\alpha \mathbf{u}_\alpha dV + \int_{\partial\Omega} \rho_\alpha \mathbf{u}_\alpha \left(\mathbf{u}_\alpha \cdot \mathbf{n}\right) dA = \int_\Omega \left(\rho_\alpha \mathbf{f}_\alpha + \mathbf{m}_\alpha\right) dV + \int_{\partial\Omega} \boldsymbol{\sigma}_\alpha \mathbf{n} dA \tag{10}$$

Following a procedure analogous to the one used to get the local form of the mass conservation equation, the linear momentum conservation equation local form for each constituent ($\alpha = 1, n$) is obtained,

$$\rho_\alpha \left[\frac{\partial \mathbf{u}_\alpha}{\partial t} + \left(\nabla \mathbf{u}_\alpha\right)\mathbf{u}_\alpha\right] = \nabla \cdot \boldsymbol{\sigma}_\alpha + \mathbf{m}_\alpha + \rho_\alpha \mathbf{f}_\alpha \tag{11}$$

Assuming the validity of equation (11), a local form of conservation equation of linear momentum for the whole mixture may be stated as $\sum_{\alpha=1}^{n} \mathbf{m}_\alpha = 0$.

Although the mixture theory only requires that the summation of all partial stress tensors be symmetric, in this work each component of the partial stress tensor $\boldsymbol{\sigma}_\alpha$ is considered as symmetric, automatically satisfying the angular momentum conservation equation.

Let e_α represent each constituent internal energy, r_α its energy generation per unit mass, $\mathbf{q}_\alpha$ the partial heat flux per unit time and unit area - associated to the α-th constituent in such a way that $\mathbf{q}_\alpha \cdot \mathbf{n}$ represents the conduction heat flux to the above mentioned constituent through the surface $\partial\Omega$. Also defining ψ_a as the (internal) energy generation term, which represents the energy per unit time and unit volume supplied to the α-th constituent due to its thermal interaction with the remaining constituents of the mixture, the conservation of energy for each constituent ($\alpha = 1, n$) may be postulated as

$$\begin{aligned}&\frac{d}{dt}\int_\Omega \rho_\alpha \left[e_\alpha + \frac{1}{2}\mathbf{u}_\alpha \cdot \mathbf{u}_\alpha\right] dV + \\ &\int_{\partial\Omega} \rho_\alpha \left[e_\alpha + \frac{1}{2}\mathbf{u}_\alpha \cdot \mathbf{u}_\alpha\right] \mathbf{u}_\alpha \cdot \mathbf{n} dA = \\ &\int_\Omega \psi_\alpha dV + \int_\Omega \left[\rho_\alpha r_\alpha + \left(\rho_\alpha \mathbf{f}_\alpha + \mathbf{m}_\alpha\right) \cdot \mathbf{u}_\alpha\right] dV \\ &+ \int_{\partial\Omega} \left[\mathbf{t}_\alpha \cdot \mathbf{u}_\alpha - \mathbf{q}_\alpha \cdot \mathbf{n}\right] dA\end{aligned} \tag{12}$$

in which $\frac{1}{2}\left(\mathbf{u}_\alpha \cdot \mathbf{u}_\alpha\right)$ represents the α-th constituent kinetic energy per unit mass, $\rho_\alpha \mathbf{f}_\alpha \cdot \mathbf{u}_\alpha$ and $\mathbf{m}_\alpha \cdot \mathbf{u}_\alpha$ represent, respectively, the power of body forces and interaction forces per unit volume and $\mathbf{t}_\alpha \cdot \mathbf{u}_\alpha$ stands for the power of surface forces per unit area.

Once again, a procedure analogous to that used before, leads to the local form of the energy equation for each constituent, for $\alpha = 1, n$

$$\rho_\alpha \left[\frac{\partial e_\alpha}{\partial t} + \left(\nabla e_\alpha\right)\mathbf{u}_\alpha\right] = \rho_\alpha r_\alpha - \nabla \cdot \mathbf{q}_\alpha + \psi_\alpha + \boldsymbol{\sigma}_\alpha \cdot \boldsymbol{\varepsilon}\left(u_\alpha\right) \tag{13}$$

with $\boldsymbol{\varepsilon}(\mathbf{u}_\alpha)$ representing the symmetrical part of $\nabla \mathrm{u}_\alpha$.

According to Martins-Costa *et al.* (1993) the conservation of energy for the mixture as a whole, in the absence of mass generation and considering $\boldsymbol{\sigma}_\alpha$ symmetrical, provided that Eq. (13) is verified, may be stated as $\sum_{\alpha=1}^{n} \psi_\alpha = 0$.

For a given material there must exist some relation between its dynamic and kinematic states at some instant and, in some cases, the material behavior at the present instant is also influenced by its kinematic state at all instants of its past history. The equations expressing the relation between kinematic and dynamic variables are called *constitutive equations*. In the solid-fluid mixture considered in this work it suffices to solve balances of mass and linear momentum for the fluid constituent which, in turn, require constitutive assumptions for the partial stress tensor $\boldsymbol{\sigma}_f$ and diffusive force $\mathbf{m}_f$. The partial stress tensor for the fluid constituent may be stated as

$$\boldsymbol{\sigma}_f = -\varphi p \mathbf{I} + 2\eta \boldsymbol{\varepsilon}(\mathbf{u}_f) \tag{14}$$

in which $\mathbf{u}_f$ and $p_f = \varphi p$ are the velocity and partial pressure fields of the fluid constituent (p being the pressure acting on the mixture), respectively, η represents a parameter related to the viscosity and $\boldsymbol{\varepsilon}(\mathbf{u}_f)$ is the symmetrical part of the fluid constituent velocity gradient. Sampaio and Williams (1977) have analyzed the case of mixtures of fluids, concluding that the parameter η was represented by the product of the fluid fraction φ and the viscosity of the base fluid μ. Williams (1978) proposed the existence of an always positive scalar parameter λ for solid-fluid mixtures - representing a fluid flow through a porous matrix. The λ-parameter depends on the solid constituent's parameters only and takes into account the porous matrix microstructure. In short, $\eta = \lambda \varphi^2 \mu$ with μ representing the fluid viscosity.

Since the porous medium is saturated by the fluid and at rest, the interaction force is proportional to $\mathbf{u}_f$ (Williams, 1978; Costa-Mattos *et al.*, 1995). The proportionality parameter is obtained by considering a limit case in which Darcy's law assumptions are valid - an infinite cross section (rigid, homogeneous and isotropic) porous medium saturated by a steady-state fluid flow having only the gradient of pressure as driving force. In this particular case the following equation describes the fluid motion,

$$\varphi \mathbf{u}_f = -\frac{K}{\mu}\nabla p \tag{15}$$

in which K represents the specific permeability of the porous medium – a scalar, since the porous matrix is assumed isotropic. Thus, the interaction force may be expressed by the following constitutive law:

$$\mathbf{m}_f = -\frac{\varphi^2 \mu}{K}\mathbf{u}_f \tag{16}$$

The partial heat fluxes for the solid ($\mathbf{q}_s$) and the fluid ($\mathbf{q}_f$) constituents are postulated by making an analogy with both Fourier's law for single continua and the

partial heat fluxes for mixtures of *n* solids considered by Saldanha da Gama (1989), as (Martins-Costa *et al.*, 1992; Martins-Costa and Saldanha da Gama, 1996)

$$\begin{aligned} \mathbf{q}_s &= -\Lambda k_S(1-\varphi)\nabla\theta_s \\ \mathbf{q}_f &= -\Lambda k_F\varphi\,\nabla\theta_f \end{aligned} \tag{17}$$

where Λ represents an always positive parameter which may depend on both the internal structure and the kinematics of the mixture and k_S and k_F are the solid and the fluid thermal conductivity (regarded from a Continuum Mechanics viewpoint).

The energy generation function, ψ_α, which is an internal contribution, represents the energy supply to a given constituent, arising from its thermal interaction with the remaining constituents of the mixture. The ψ_α function is zero at a given point only if all the constituents are at the same temperature at this point (Saldanha da Gama, 1989; Martins Costa *et al.*, 1992; Martins-Costa and Saldanha da Gama, 1996). The coefficients R_{sf} and R_{fs} are positive valued parameters which depend on both constituent's thermal properties, on the mixture internal structure and on the fluid constituent velocity, since the solid constituent is supposed to be at rest. In order to satisfy the energy balance for the mixture (Martins-Costa *et al.*, 1993), which states that in the absence of mass generation and assuming the partial stress tensor symmetrical the sum $\psi_f + \psi_s$ must be zero, it is necessary to ensure that $R_{sf} \equiv R_{fs} \equiv R$, so the internal energy generation may be described by

$$-\psi_s = \psi_f = R(\theta_s - \theta_f) \tag{18}$$

III. FINITE ELEMENT MODELING

Substituting the constitutive Eqs. (14)-(18) into the conservation Eqs. (11) and (13) and assuming low velocities and steady-state regime, the following boundary-value problem may be obtained for incompressible thermal flows through saturated porous media: *Given functions* $\mathbf{f}_f : \overline{\Omega} \to \mathbf{R}^2$, $r_f : \overline{\Omega} \to \mathbf{R}$ *and* $r_s : \overline{\Omega} \to \mathbf{R}$, *find the unknown fields* $\mathbf{u}_f : \overline{\Omega} \to \mathbf{R}^2$, $p : \overline{\Omega} \to \mathbf{R}$, $\theta_f : \overline{\Omega} \to \mathbf{R}$ *and* $\theta_s : \overline{\Omega} \to \mathbf{R}$, *such that*

$$\begin{aligned} -2\lambda\varphi\mu\nabla\cdot\boldsymbol{\varepsilon}(\mathbf{u}_f) + \nabla p + \frac{\varphi\mu}{K}\mathbf{u}_f &= \rho\mathbf{f}_f && \text{in } \Omega \\ \nabla\cdot\mathbf{u}_f &= 0 && \text{in } \Omega \\ \mathbf{u}_f\cdot\nabla\theta_f - \kappa_f\Delta\theta_f + \beta(\theta_f-\theta_s) &= r_f && \text{in } \Omega \\ \kappa_s\Delta\theta_s + (\theta_s-\theta_f) &= r_s && \text{in } \Omega \\ \mathbf{u}_f &= \mathbf{u}_g && \text{on } \Gamma_g \\ \theta_f &= \theta_{fg} && \text{on } \Gamma_g \\ \theta_s &= \theta_{sg} && \text{on } \Gamma_g \\ \boldsymbol{\sigma}_f\mathbf{n} &= \boldsymbol{\sigma}h && \text{on } \Gamma_h \\ -\Lambda k_f\varphi\nabla\theta_f\cdot\mathbf{n} &= q_{fh} && \text{on } \Gamma_h \\ -\Lambda k_s(1-\varphi)\nabla\theta_s\cdot\mathbf{n} &= q_{sh} && \text{on } \Gamma h \end{aligned} \tag{19}$$

where Γ_g is the region of the boundary Γ on which essential (Dirichlet) conditions are imposed and Γ_h is subjected to the natural (Neumman) ones. Also, $\mathbf{u}_f$ represents the fluid constituent velocity, $\boldsymbol{\sigma}_f$ the partial stress tensor acting on it (Eq. (14)), $\boldsymbol{\varepsilon}(\mathbf{u}_f)$ the symmetrical part of the $\nabla\mathbf{u}_f$-tensor and $\mathbf{f}_f$ the gravitational body force. The parameter φ is the fluid fraction, p is the pressure acting on the mixture, μ is the fluid viscosity and K is the porous matrix porosity, both regarded from a Continuum Mechanics viewpoint. The thermal porous diffusivities κ_f and κ_s and coefficient β defined, respectively, by

$$\kappa_f = \frac{\Lambda k_f\varphi}{\rho_f c_f};\quad \beta = \frac{R}{\rho_f c_f};\quad \kappa_s = \frac{\Lambda k_s(1-\varphi)}{R} \tag{20}$$

and prescribed energy supplies r_f and r_s redefined as

$$r_f = \frac{r_f}{\rho_f c_f};\quad r_s = \frac{r_s}{R} \tag{21}$$

Considering the simple geometry depicted in Figure 2 the hydrodynamic part of the problem has an analytical solution as defined in Eq. (15) (for details, see Martins-Costa *et al.*, 1992). So, a finite element approximation will be constructed for the advective-diffusive heat transfer boundary-value problem defined in Eqs. (19)-(21), based on the following finite dimension subspaces,

$$\begin{aligned} W_h &= \{\phi \in H_0^1(\Omega)\,\big|\,\phi_{|K} \in P_m(K), K \in C_h\} \\ W_h^g &= \{\phi \in H^1(\Omega)\,\big|\,\phi_{|K} \in P_m(K), \\ &\quad K \in C_h, \phi = \theta_{ig} \text{ on } \Gamma_g\} \quad i = f, s \end{aligned} \tag{22}$$

where P_m denotes a polynomial space of degree m, C_h is a partition of $\overline{\Omega}$ into elements consisting of convex quadrilaterals, performed in the usual way (Ciarlet, 1978), $H^1(\Omega)$ is the Sobolev space of functions and 1st-derivatives with square-integrable value in Ω and $H_0^1(\Omega)$ is the Sobolev space of functions with square-integrable value and 1st-derivatives in Ω which vanishes on Γ.

In order to approximate the thermal problem of system (19)-(21), employing the space definitions (22), we have made use of the following Petrov-Galerkin formulation: *Find the pair* $(\theta_{fh}, \theta_{sh}) \in W_h^g \times W_h^g$ *such that*

$$B(\theta_{fh}, \theta_{sh}; \phi_f, \phi_s) = F(\phi_f, \phi_s),\ \ (\phi_f, \phi_s) \in W_h \times W_h \tag{23}$$

with

$$\begin{aligned} B(\theta_{fh}, \theta_{sh}; \phi_f, \phi_s) &= (\mathbf{u}_f\cdot\nabla\theta_f, \phi_f) \\ &+(\kappa_f\nabla\theta_f, \nabla\phi_f) + (\beta(\theta_f-\theta_s), \phi_f) \\ &+(\kappa_s\nabla\theta_s, \nabla\phi_s) + ((\theta_s-\theta_f), \phi_s) \\ &+\sum\nolimits_{K\in C_h}(\mathbf{u}_f\cdot\nabla\theta_f - \kappa_f\Delta\theta_f + \beta(\theta_f-\theta_s), \\ &\quad \tau(Pe_K)(\mathbf{u}_f\cdot\nabla\phi_f - \kappa_f\Delta\phi_f))_K \end{aligned} \tag{24}$$

and

$$F(\phi_f,\phi_s)=(r_f,\phi_f)+(r_s,\phi_s)+(q_{fh},\phi_f)_h$$
$$+(q_{sh},\phi_s)_h+\sum_{K\in C_h}(r_f,\tau(Pe_K)(\mathrm{u}_f\cdot\nabla\phi_f-\kappa_f\Delta\phi_f))_K \quad (25)$$

in which the fluid constituent velocity $\mathbf{u}_f$ has an analytical solution as defined in Eq. (28) and the parameter τ is based upon the stability parameter introduced in Franca *et al.* (1992) for the scalar context of the advective-diffusive equation:

$$\tau(\mathrm{Pe}_K) = \frac{h_K}{2\left|\mathbf{u}_f\right|_p}\ \xi(Pe_K) \quad (26)$$

$$\xi(\mathrm{Pe}_K) = \begin{cases}\mathrm{Pe}_K\ , & 0\le \mathrm{Pe}_K<1\\ 1\ , & \mathrm{Pe}_K\ge 1\end{cases} \quad (27)$$

$$\mathrm{Pe}_K = \frac{mk\,|\,\mathbf{u}_f\,|_p\ h_K}{2\kappa_f} \quad (28)$$

$$m_k=\min\left\{\frac{1}{3},\ 2C_k\right\} \quad (29)$$

$$\left|\mathbf{u}_f(x)\right|_p=\begin{cases}\left(\sum_{i=1}^{N}\left|u_i(\mathbf{x})^p\right|\right)^{1/p}, & 1\le p<\infty\\ \max\limits_{i=1,N}\left|u_i(\mathbf{x})\right|\ , & p=\infty\end{cases} \quad (30)$$

$$C_k\sum_{K\in C_h}h_K^2\left\|\Delta\phi_f\right\|_{0,K}^2\le\left\|\Delta\phi_f\right\|_0^2\quad \phi_f\in W_h \quad (31)$$

where C_k is a positive constant independent of C_h.

Remark 1: If we drop the mesh-dependent terms from the above formulation, i.e. set $\tau = 0$ throughout, then the method is reduced to the classical Galerkin method for the thermal problem defined in system (19): *Find the pair* $(\theta_{fh},\theta_{sh})\in W_h^g\times W_h^g$ *such that*

$$B_G(\theta_{fh},\theta_{sh};\phi_f,\phi_s)=F_G(\phi_f,\phi_s),$$
$$(\phi_f,\phi_s)\in W_h\times W_h \quad (32)$$

with

$$B_G(\theta_{fh},\theta_{sh};\phi_f,\phi s)=(\mathbf{u}_f\cdot\nabla\theta_f,\phi_f)+$$
$$(\kappa_f\nabla\theta_f,\nabla\phi_f)+(\beta(\theta_f-\theta_s),\phi_f) \quad (33)$$
$$+(\kappa_s\nabla\theta_s,\nabla\phi_s)+((\theta_s-\theta_f),\ \phi_s)$$

and

$$F_G(\phi_f,\phi_s)=(r_f,\phi_f)+(r_s,\phi_s)+(q_{fh},\phi_f)_h$$
$$+(q_{sh},\phi_s)_h \quad (34)$$

Remark 2: Accordingly Franca *et al*. (1992), the usual Péclet grid number (Johnson, 1987) was modified by including the parameter m_k in eq.(29), to account for the degree of interpolation employed. As a consequence advective-dominated flow regions are characterized by $\mathrm{Pe}_K > 1$ and diffusive-dominated ones by $\mathrm{Pe}_K < 1$, regardless the element considered.

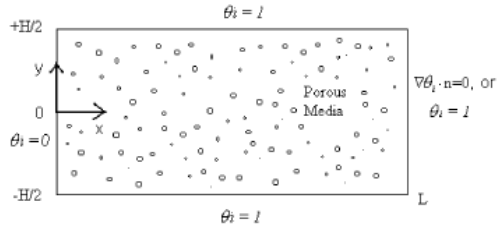

Figure 2. Flow into a porous channel: Problem statement.

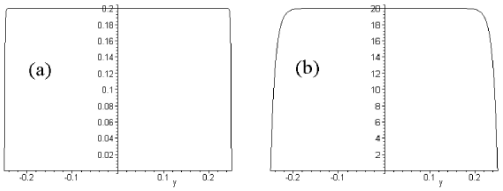

Figure 3. Fluid constituent velocity profiles: (a) for $Da = 4\times$ 10-6; (b) for $Da = 4\times$ 10-4.

IV. NUM ERICAL RESULTS

In this section, numerical simulations of the thermal problem defined in Eqs. (19)-(21) employing the Petrov-Galerkin formulation defined by Eqs. (23)-(31) are presented. Lagrangean bilinear (Q1) and Serendipity biquadratic (Q2S) interpolations have been utilized to approximate the temperature fields of fluid and solid constituents. All computations performed in this article have been carried out at Laboratory of Computational and Applied Fluid Mechanics (LAMAC) of Federal University of Rio Grande do Sul, making use of the finite element code FEM and graphic post-processor VIEW - both codes under development in the above mentioned Laboratory.

The studied geometry is illustrated in Fig. 2: a newtonian fluid flows through a horizontal porous channel limited by impermeable and isothermal flat plates. Considering the classical no-slip condition on the impermeable .at plates,

$$\mathbf{u}_f=0\quad \text{for } 0<x<L,\quad y=\pm H/2 \quad (35)$$

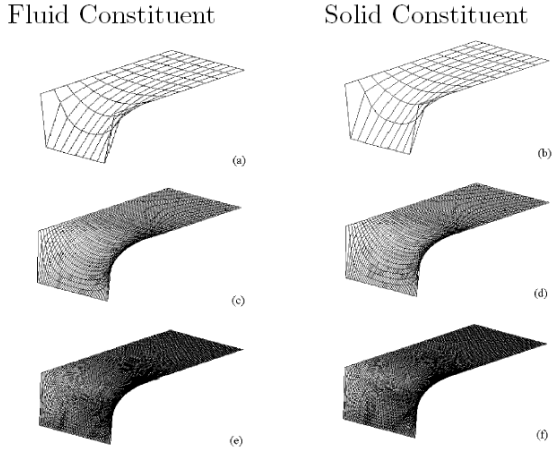

Figure 4. Temperature distributions for low Péclet numbers with Q1 elements: Elevation plots. (a) fluid constituent (mesh 10x10); (b) solid constituent (mesh 10x10); (c) fluid constituent (mesh 50x50); (d) solid constituent (mesh 50x50);

(e) fluid constituent (mesh 100x100); (f) solid constituent (mesh 100x100).

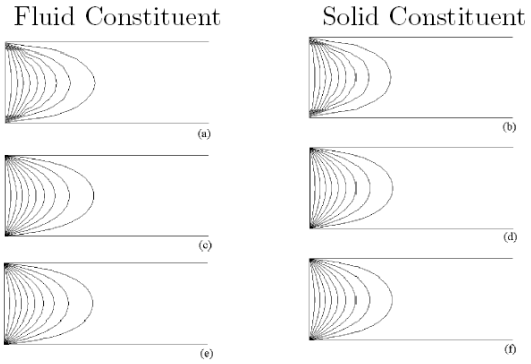

Figure 5. Temperature distributions for low Péclet flow with Q1 elements: Contours. (a) fluid constituent (mesh 10x10); (b) solid constituent (mesh 10x10); (c) fluid constituent (mesh 50x50); (d) solid constituent (mesh 50x50); (e) fluid constituent (mesh 100x100); (f) solid constituent (mesh 100x100).

an analytical velocity profile has been obtained for the fluid constituent (Martins-Costa *et al*., 1992) by solving the hydrodynamic problem associated to Eqs. (19)-(21), giving rise to

$$\mathbf{u}_f(y) = \frac{K}{\mu\varphi}\left[\rho\mathbf{f}_{fx} - \frac{\partial p}{\partial x}\right]\left[1 - \frac{\cosh\dfrac{y}{\sqrt{K\lambda}}}{\cosh\dfrac{H/2}{\sqrt{K\lambda}}}\right] \qquad (36)$$

with $-H/2 < y < H/2$, in which u_f and $\mathbf{f}_{fx}$ represent, respectively, the x-component of $\mathbf{u}_f$ and the gravitational body field $\mathbf{f}_f$.

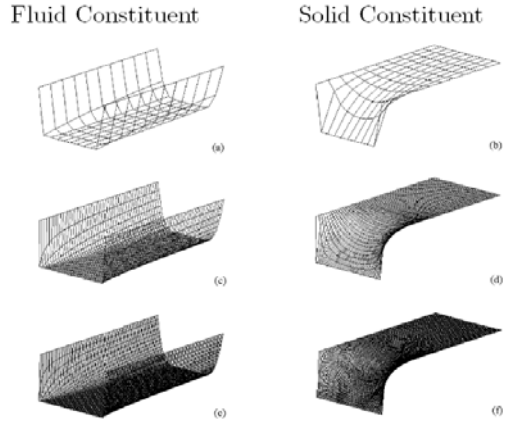

Figure 6. Temperature distributions for high Péclet flow with Q1 elements: Elevation plots. (a) fluid constituent (mesh 10x10); (b) solid constituent (mesh 10x10); (c) fluid constituent (mesh 50x50); (d) solid constituent (mesh 50x50); (e) fluid constituent (mesh 100x100); (f) solid constituent (mesh 100x100).

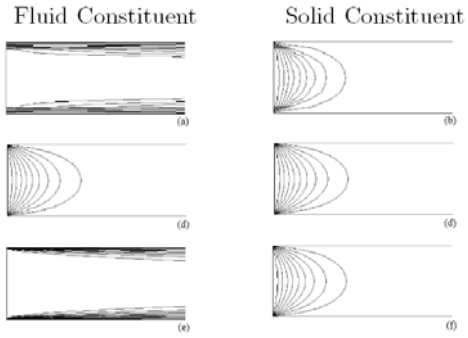

Figure 7. Temperature distributions for high Péclet flow with Q1 elements: Contours. (a) fluid constituent (mesh 10x10); (b) solid constituent (mesh 10x10); (c) fluid constituent (mesh 50x50); (d) solid constituent (mesh 50x50); (e) fluid constituent (mesh 100x100); (f) solid constituent (mesh 100x100).

At this point it is interesting to mention that for the limiting case in which $H \to \infty$, the velocity given by Eq.(36) approaches the one obtained from the classical Darcy's law, given by

$$v_{Darcy} = \frac{K}{\mu\varphi}\left[\rho \mathrm{f}_{fx} - \frac{\partial p}{\partial x}\right] \qquad (37)$$

The almost flat velocity profile depicted in Fig. 3 was obtained from Eq. (28), approaching the classical Darcy's law velocity expression (Bejan, 1987) as the channel width $H \to \infty$. Figure 3 compares velocity profiles for two distinct values of Darcy number – relating the porous matrix permeability to the channel width ($Da = K/H^2$), namely Da = 4 × 10^{-4} and Da= 4 × 10^{-6}, showing that the velocity profile could be described by the classical Darcy's law for small Darcy numbers.

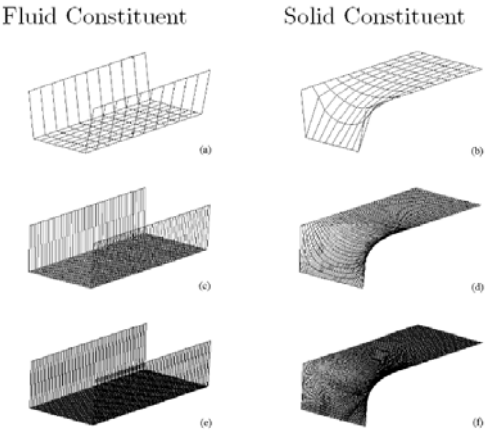

Figure 8. Temperature distributions for very high Péclet flow with Q1 elements: Elevation plots.(a) fluid constituent (mesh 10x10); (b) solid constituent (mesh 10x10); (c) fluid constituent (mesh 50x50); (d) solid constituent (mesh 50x50); (e) fluid constituent (mesh 100x100); (f) solid constituent (mesh 100x100).

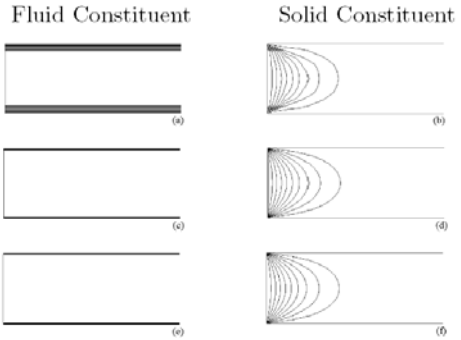

Figure 9. Temperature distributions for very high Péclet flow with Q1 elements: Contours.(a) fluid constituent (mesh 10x10); (b) solid constituent (mesh 10x10); (c) fluid constituent (mesh 50x50); (d) solid constituent (mesh 50x50); (e) fluid constituent (mesh 100x100); (f) solid constituent (mesh 100x100).

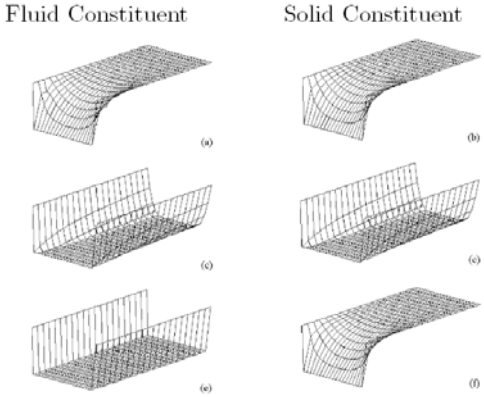

Figure 10. Temperature distributions for a 10x10 mesh with Q2S elements: Elevation plots.(a) fluid constituent (low Péclet number); (b) solid constituent (low Péclet number); (c) fluid constituent (high Péclet number); (d) solid constituent (high Péclet number); (e) fluid constituent (very high Péclet number); (f) solid constituent (very high Péclet number).

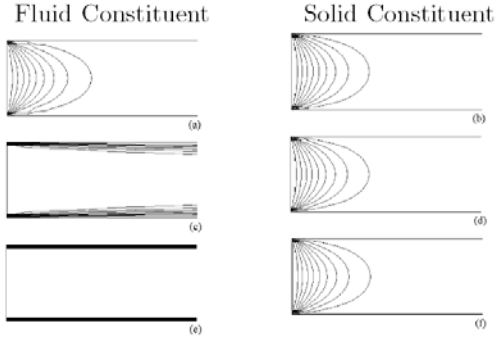

Figure 11. Temperature distributions for a 10x10 mesh with Q2S elements: Contours.(a) fluid constituent (low Péclet number); (b) solid constituent (low Péclet number); (c) fluid constituent (high Péclet number); (d) solid constituent (high Péclet number); (e) fluid constituent (very high Péclet number); (f) solid constituent (very high Péclet number).

A flat porous channel of aspect ratio $L/H = 2$ (with L denoting its length and H its width), thermal porous diffusivities $\kappa_f = 10^0$, 10^{-3} and 10^{-7} for the fluid constituent and $\kappa_s = 10^1$ for the solid one and the β-coefficient assuming the value $\beta = 10^{-2}$ has been simulated, considering the following boundary conditions

$$\begin{cases} \theta_i = 1 & 0 < x < L,\ y = \pm H/2 \\ \theta_i = 0 & x = 0,\ -H/2 < y < H/2 \\ \theta_i = 1 \text{ or } \nabla\theta_i \cdot \mathbf{n} = 0 & x = L,\ -H/2 < y < H/2 \end{cases} \quad (38)$$

where **n** is the unit outward normal vector.

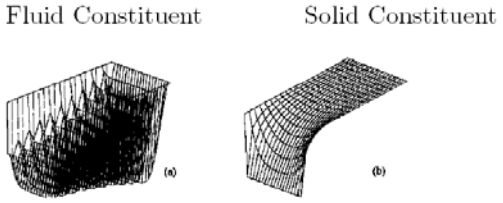

Figure 12. Galerkin method employing Q2S elements with Dirichlet conditions at the outlet: Elevation plots. (a) fluid constituent (b) solid constituent

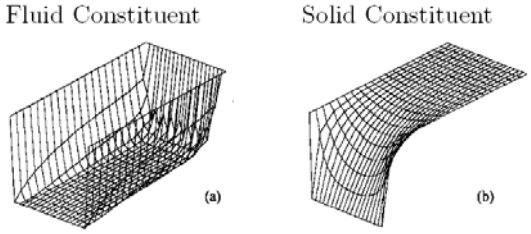

Figure 13. Petrov-Galerkin method employing Q2S elements with Dirichlet conditions at the outlet: Elevation plots. (a) fluid constituent (b) solid constituent

Taking the centerline velocity as the characteristic flow velocity and fixing the channel width as $H = 0.5$, the following porous Péclet numbers are obtained:

$$\square \textit{for } \kappa_F = 1,\ \overline{\mathrm{Pe}}^{I} = v_F(0)H/\kappa_F = 5\times10^{-1};$$

$$\square \textit{for } \kappa_F = 10\text{-}3,\ \overline{\mathrm{Pe}}^{II} = 5\times10^{2};$$

$$\square \textit{for } \kappa_F = 10\text{-}7,\ \overline{\mathrm{Pe}}^{III} = 5\times10^{6}.$$

Figures 4 to 9 illustrate the Petrov-Galerkin approximation defined in Eq. (23)-(31) for three different meshes (10x10, 50x50 and 100x100) employing lagrangean bilinear elements (Q1) for a large range of Péclet flows. (The channel geometry was fixed at $L/H = 2$.). The first flow, depicted in Fig. 4 and 5, has a porous thermal diffusivity $\kappa_F = 1$ providing a diffusive dominated flow; in Fig. 6 and 7, a high advective dominated one is illustrated, namely $\kappa_F = 10^{-3}$, while in Fig. 8 and 9 a very high advective flow is shown, since κ_F is equal to 10^{-7}. In all situations the employed Petrov-Galerkin method has approximated stably θ_f and θ_s temperature fields even for the sharpest ones, namely, for θ_f field with $\kappa_F = 10^{-3}$ and $\kappa_F = 10^{-7}$. In the former, shown in Fig. 6 and 7, a global smooth pattern with small oscillations localized in the boundary layers at channel walls may be noticed; in the latter, an excellent resolution is achieved without oscillations even in for very thin thermal boundary layers as those generated in Fig. 8 and 9.

Figures 10 and 11 illustrate the Petrov-Galerkin approximation with a 10x10 serendipity biquadratic (Q2S) elements, considering all the flows described in the last paragraph – namely diffusive, high advective and very high advective ones. It is worth to be noticed that even for a poor mesh with only 10x10 elements, the

method defined by Eqs. (23)-(31) was able to obtain an excellent resolution inside and outside the boundary layers.

In Fig. 12 and 13 elevation plots for $\kappa_F = 10^{-3}$ are shown employing the Galerkin and the Petrov-Galerkin methods defined, respectively, in Eqs. (32)-(34) and (23)-(31), with a Dirichlet outflow boundary condition. Since the flow is parallel to the mesh, one might conjecture that the Galerkin formulation might work, which was not confirmed herein, while the Petrov-Galerkin one points out an excellent pattern with only small oscillations near the channel exit. Galerkin method poor performance is due to the outflow condition employed that creates an outflow boundary layer, which wholly contaminates Galerkin approximation for the fluid constituent.

Eventually, it is of worth to be marked that all θ_f and θ_s distributions shown are plainly smooth indicating that the numerical method employed was capable of generating stable and accurate temperature approximations even for high Péclet flows, $10^3 < \mathrm{Pe}_K < 10^7$.

V. FINAL REMARKS

In this work a mechanical modeling for a non-classical approach for the heat transfer in a flow through a porous channel has been presented. This model, which is based on the continuum theory of mixtures, generates a hydrodynamic problem analogous to the steady-state Stokes problem (considering the geometry depicted in Fig. 2) – except for the momentum source term – which is used as input for the thermal problem. The latter is a boundary value problem consisting of two coupled partial equations, one describing the fluid constituent energy balance and the other the solid one. (The coupling term is an energy source term.) A Petrov-Galerkin formulation has been employed to approximate the variational equations describing the non-isothermal advective flow.

For low Péclet regime, the results have pointed out thermal equilibrium between both constituents, an expected behavior since the energy source term $\beta(\theta_f - \theta_s)$ is not overwhelmed by the advective one in the fluid energy balance equation. For high and very high Péclet flows, stabilization strategies proved to be essential, being able to capture accurately smooth thermal boundary-layers near the channel walls (for high Péclet flows) and, for very high Péclet flows, the Petrov-Galerkin formulation simulated a quasi-hyperbolic exact solution. At length, when classical Galerkin and Petrov-Galerkin formulations are subjected to Dirichlet outflow boundary conditions, it can be clearly noticed the Galerkin fails drastically to simulate advective-dominated regime flows.

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