INFLUENCE OF MAGNETIC FIELD ON NATURAL CONVECTION FLOW NEAR A WAVY CONE IN POROUS MEDIA

A. MAHDY[†*] , R.A. MOHAMED[†] and F.M. HADY[‡]

† *Math. Department, Science, South Valley University, Qena, EGYPT*
[*] *sci_eng@yahoo.com*
‡ *Math. Department, Science, Assiut University, EGYPT*

***Abstract*— The results of a study of effect of magnetic field on natural convection along an isothermal wavy cone embedded in a fluid-saturated porous medium are obtained. A coordinate transformation is used to transform the complex wavy conical surface to a smooth conical surface, and the transformed non-similar boundary layer governing equations are then solved numerically by means of the Runge-Kutta integration scheme with the Newton-Raphson shooting method. The boundary layer under consideration is concerned with the regime where the Darcy-Rayleigh number *Ra* is very large. Detailed results of the effect of magnetic field, half cone angle, and the sinusoidal wavy surface on the velocity, temperature and the wall heat flux are presented.**

***Keywords*— Magnetic field, wavy cone, porous media.**

I. INTRODUCTION

The study and analysis of heat transfer in porous media have been the subject of many investigations due to their frequent occurrence in industrial and technological applications. Examples of some applications include geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, and many others. There has been considerable work done on free convection flow over conical surfaces which is based on the laminar boundary-layer approach. See, for instance, Cheng and Minkowycz (1977), Chamkha (1996), Kafoussias (1992), Yih (1999), Alamgir (1979), Na and Chiou (1979a, b), El-kabeir *et al.* (2006), Hering and Grosh (1965) and Roy (1974), and many other papers can be found in Nield and Bejan (1999). Most early studies on convection heat transfer in porous media have used regular surfaces.

The study of heat transfer near irregular surfaces is of fundamental importance; that is because it is often met in many practical applications and devices such as flat-plate solar collectors and flat-plate condensers in refrigerators. The presence of roughness elements disturbs the flow past surfaces and alters the heat transfer rate. Yao (1983) studied the natural convection heat transfer from isothermal vertical wavy surfaces, such as sinusoidal surfaces, in Newtonian fluids. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces. Rees and Pop (1994a, b) and (1995) carried out some studies to analyze natural convection from vertical and horizontal wavy surfaces embedded in a porous medium. Hady *et al.* (2006) analyzed the problem of MHD free convection flow along a vertical wavy surface in presence of magnetic field and generation absorption. Pop and Na (1994) and (1995) studied the natural convection flow along a vertical wavy cone and a frustum of a wavy cone in porous media. Hossain and Pop (1996) studied the magnetohydrodynamic boundary layer flow and heat transfer on a continuous moving wavy surface in Newtonian fluids. Cheng (2000a, b) and Cheng (2007) reported the phenomenon of natural convection heat and mass transfer near a vertical wavy surface and near a wavy cone and a frustum wavy cone with constant wall temperature and concentration in a porous medium.

Motivated by the works mentioned above, the steady, laminar, free convection flow along a wavy cone and immersed in an electrically conducting fluid-saturated porous medium in the presence of a transverse magnetic field is considered. The surface temperature of the cone is assumed to be constant. The applied magnetic field is assumed to be uniform and the magnetic Reynold's number is assumed to be small so that the induced magnetic field can be neglected. In addition, it is assumed that the external electric field is zero and the electric field due to polarization of charges is negligible.

II. MATHEMATICAL ANALYSIS

Consider the steady-state boundary layer flow near a wavy cone with transverse sinusoidal undulations embedded in a fluid-saturated porous medium, as illustrated in Fig. (1). The wavy surface profile is given by

$$\hat{y} = \hat{\delta} = \hat{a}\sin(\pi\,\hat{x}/\ell), \tag{1}$$

where a is the amplitude of the wavy surface, ℓ is the length scale of the wavy surface. The origin of the coordinate system is placed at the leading edge of the wavy cone. We assume that the temperature of the wavy cone is held at constant value T_w and a uniform ambient temperature T_∞. The fluid properties are assumed to be constant except for density variations in the buoyancy force term.

Based on Boussinesq approximations, the equations governing the steady-state conservation of mass, momentum, and energy for Darcy flow through a homogeneous porous medium near the wavy cone can be given as

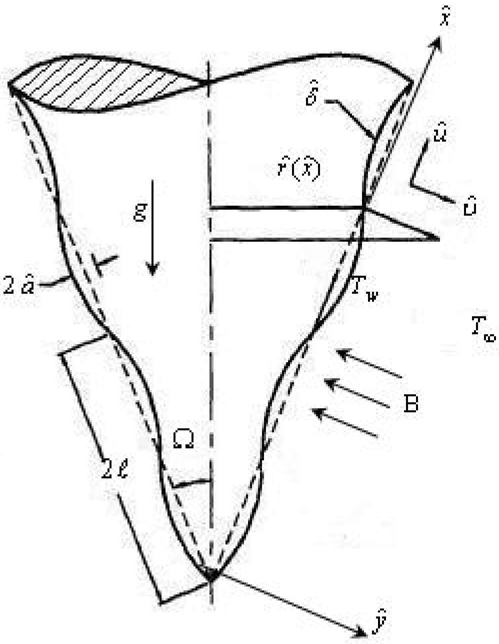

Fig. 1. Physical model and coordinates.

$$\frac{\partial(\hat{r}\,\hat{u})}{\partial\hat{x}} + \frac{\partial(\hat{r}\,\hat{\upsilon})}{\partial\hat{y}} = 0, \tag{2}$$

$$\frac{\partial\hat{u}}{\partial\hat{y}} - \frac{\partial\hat{\upsilon}}{\partial\hat{x}} = \frac{K\,g\,\beta}{\nu}\left(\cos\Omega\,\frac{\partial T}{\partial\hat{y}} + \sin\Omega\,\frac{\partial T}{\partial\hat{x}}\right) - \frac{K\sigma B^2}{\mu}\frac{\partial\hat{u}}{\partial\hat{y}} \tag{3}$$

$$\hat{u}\frac{\partial T}{\partial\hat{x}} + \hat{\upsilon}\frac{\partial T}{\partial\hat{y}} = \alpha\left(\frac{\partial^2 T}{\partial\hat{x}^2} + \frac{\partial^2 T}{\partial\hat{y}^2}\right). \tag{4}$$

The boundary conditions to be considered are:

$$\begin{aligned} &\hat{y} = \hat{\delta}(\hat{x}): && \hat{\upsilon} = 0, && T = T_w, \\ &\hat{y}\to\infty: && \hat{u} = 0, && T = T_\infty, \end{aligned} \tag{5}$$

where $\hat{u}$ and $\hat{\upsilon}$ are the volume-averaged velocity components in the $\hat{x}$ and $\hat{y}$ directions respectively. T is the volume-averaged temperature; Ω and r are the half angle and the local radius of the smooth surface of the cone. α, β are the thermal diffusivity of the saturated porous medium and thermal expansion coefficients of the fluid; K is the permeability of the porous medium. Properties ν and μ are the effective kinematic viscosity, and dynamic viscosity of the fluid, respectively, g is the acceleration due to gravity, σ, B are the electrical conductivity and the applied magnetic flux density.

Because the boundary layer thickness is small, the local radius to a point in the boundary layer $\hat{r}(\hat{x})$ can be represented by the radius of the cone,

$$\hat{r} = \hat{x}\sin\Omega.$$

Introducing the stream function $\hat{\Psi}$ defined by

$$\hat{u} = \frac{1}{\hat{r}}\frac{\partial\hat{\Psi}}{\partial\hat{y}}, \qquad \hat{\upsilon} = = -\frac{1}{\hat{r}}\frac{\partial\hat{\Psi}}{\partial\hat{x}},$$

with the following dimensionaless variables:

$$x^* = \frac{\hat{x}}{\ell},\ y^* = \frac{\hat{y}}{\ell},\ a^* = \frac{\hat{a}}{\ell},\ \delta^* = \frac{\hat{\delta}}{\ell},\ r^* = \frac{\hat{r}}{\ell},$$

$$u^* = \frac{\hat{u}\,\ell}{\alpha},\ \upsilon^* = \frac{\hat{\upsilon}\,\ell}{\alpha},\ \Psi^* = \frac{\hat{\Psi}}{\alpha},\ \theta = \frac{T - T_\infty}{T_w - T_\infty}. \tag{6}$$

Equations (2)-(4) convert to the following equations:

$$\frac{\partial(r^*u^*)}{\partial x^*} + \frac{\partial(r^*\upsilon^*)}{\partial y^*} = 0, \tag{7}$$

$$\frac{\partial u^*}{\partial y^*} - \frac{\partial\upsilon^*}{\partial x^*} = Ra\left(\frac{\partial\theta}{\partial y^*} + \tan\Omega\frac{\partial\theta}{\partial x^*}\right) - M\frac{\partial u^*}{\partial y^*}, \tag{8}$$

$$u^*\frac{\partial\theta}{\partial x^*} + \upsilon^*\frac{\partial\theta}{\partial y^*} = \left(\frac{\partial^2\theta}{\partial x^{*2}} + \frac{\partial^2\theta}{\partial y^{*2}}\right), \tag{9}$$

where $Ra = Kg\beta\,\ell(T_w - T_\infty)\cos\Omega/\alpha\nu$ is the modified Rayleigh number, and $M = K\sigma B^2/\mu$ is the magnetic field parameter.

Equations (8) and (9) can be rewritten as follows:

$$\frac{\partial^2\Psi^*}{\partial x^{*2}} + \frac{\partial^2\Psi^*}{\partial y^{*2}} - \frac{\sin\Omega}{r^*}\left(\frac{\partial\Psi^*}{\partial x^*} + \frac{\partial\Psi^*}{\partial y^*}\right) = r^*Ra\left(\frac{\partial\theta}{\partial y^*} + \tan\Omega\frac{\partial\theta}{\partial x^*}\right) - M\frac{\partial^2\Psi^*}{\partial y^{*2}}, \tag{10}$$

$$\frac{\partial\Psi^*}{\partial y^*}\frac{\partial\theta}{\partial x^*} - \frac{\partial\Psi^*}{\partial x^*}\frac{\partial\theta}{\partial y^*} = r^*\left(\frac{\partial^2\theta}{\partial x^{*2}} + \frac{\partial^2\theta}{\partial y^{*2}}\right). \tag{11}$$

The boundary conditions are now:

$$\begin{aligned} &y^* = 0, && \Psi^* = 0, && \theta = 1, \\ &y^*\to\infty, && \partial\Psi^*/\partial y^*\to 0, && \theta\to 0. \end{aligned}$$

Now let us assume that the Darcy-Rayleigh number Ra is very large so that natural convection takes place within a boundary layer whose cross-stream width is substantially smaller than the amplitude a of the wavy surface of the cone. Accordingly, we define new variables by subtracting out the effect of the wavy surface and then introduce the usual boundary layer variables defined as:

$$x = x^*,\quad y = (y^* - \delta^*)Ra^{1/2},\quad \Psi = Ra^{-1/2}\,\Psi^*. \tag{12}$$

Substituting Eq. (12) into Eqs. (10)-(11) and $Ra\to\infty$, we get the following boundary layer equations.

$$\frac{1+\dot{\delta}^2}{r}\frac{\partial^2\Psi}{\partial y^2} = (1 - \dot{\delta}\tan\Omega)\frac{\partial\theta}{\partial y} - M\frac{\partial^2\Psi}{\partial y^2}, \tag{13}$$

$$\left(1+\dot{\delta}^2\right)\frac{\partial^2\theta}{\partial y^2} = r^{-1}\left(\frac{\partial\Psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial\theta}{\partial y}\right). \tag{14}$$

Again, we may reduce Eqs. (13)-(14) to a form more convenient for numerical solution by the transformation:

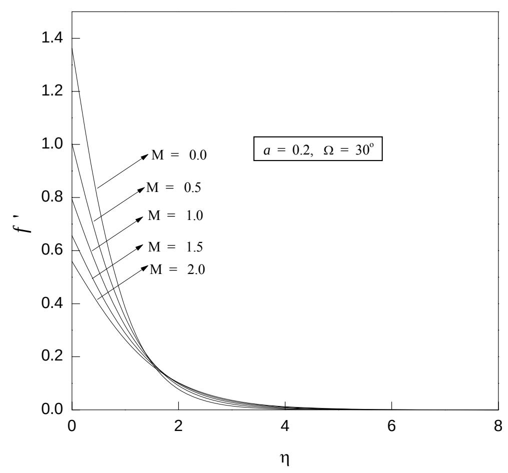

Fig. 2. Effect of M on the velocity against η, $x = 1.0$

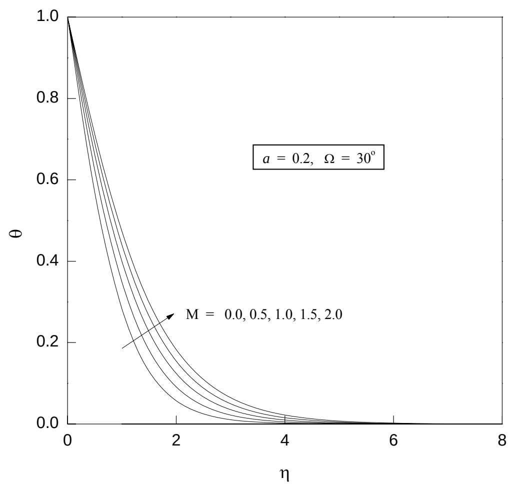

Fig. 3. Effect of M on the temperature against η, $x = 1.0$

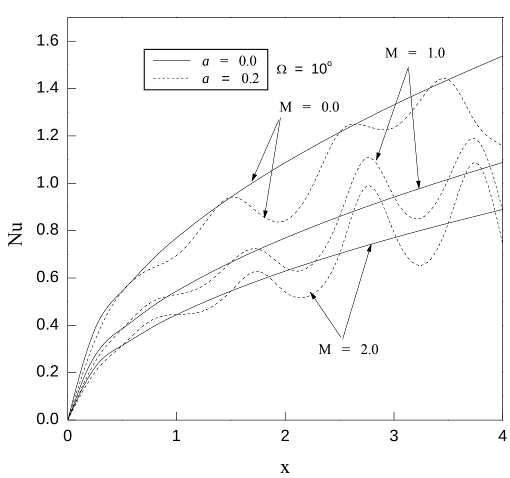

Fig. 4. Effect of M on the local Nusselt number against x

$$\eta = y(1+\dot{\delta}^2)^{-1}\, x^{-1/2}, \quad \Psi = r\, x^{1/2}\, f(x,\eta). \tag{15}$$

Substituting Eq. (15) into Eqs. (13)-(14), we obtain the following equations:

$$\left(1+\frac{M}{1+\dot{\delta}^2}\right) f'' = (1-\dot{\delta}\tan\Omega)\theta', \tag{16}$$

$$\theta'' + \frac{3}{2} f\,\theta' = x\left(f'\frac{\partial\theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right). \tag{17}$$

Note that primes denote differentiation with respect to η only, while $\dot{\delta} = d\delta/dx$.

The associate boundary conditions are:

$$\eta \quad 0, \quad f=0, \quad \theta=1, \quad \eta\to\infty, \quad f'\to 0, \ \theta\to 0. \tag{18}$$

An important physical quantity for this problem is the local Nusselt number

$$Nu = \frac{\hat{x}\,\hat{q}_w}{k\,(T_w - T_\infty)}, \tag{19}$$

where k is the effective thermal conductivity of the fluid-saturated porous medium and $\hat{q}_w$ is the local heat flux at the wall, and is defined by:

$$\hat{q}_w = -k\, n\cdot\hat{\nabla}T, \tag{20}$$

and

$$n = \left(-\frac{\dot{\delta}}{\sqrt{1+\dot{\delta}^2}}, \frac{1}{\sqrt{1+\dot{\delta}^2}}\right),$$

is the unit vector normal to the wavy surface of the cone.

Employing transformation (6), (12), (15) with (20) we get the local Nusselt number from the following expressions:

$$\frac{Nu}{Ra^{1/2}} = -\frac{x^{1/2}}{\left(1+\dot{\delta}^2\right)^{1/2}}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}. \tag{21}$$

III. RESULTS AND DISCUSSION

The transformed partial differential Eqs. (16) and (17) subject to the boundary conditions (18), are solved numerically by means of the Runge-Kutta fourth-order integration scheme with the Newton-Raphson shooting technique with a systematic guessing of $f'(x,0)$, $\theta'(x,0)$ for a range values of the governing physical parameters. In this section, a representative set of numerical results for the velocity, temperature, as well as the local Nusselt number, is presented graphically in Figs. (2) through (10). These results illustrate the effects of the amplitude-wavelength a, the magnetic field parameter M, and half angle of smooth conical Ω.

Figures (2) and (3) present typical profiles for the velocity along the cone f', and temperature θ for various values of the magnetic field parameter M, with $\Omega = 30^0$, $a = 0.2$. Application of a magnetic field normal to the flow of an electrically conducting fluid gives rise to a resistive force that acts in the direction opposite to that of the flow. This force is called the Lorentz force. This resistive force tends to slow down the motion of the fluid along the cone and causes increases in its temperature. This is depicted in Figures by the decreases in the values of f' and increases in the values of θ. Figure (4) shows the axial distribution of the heat transfer coefficient in terms of the local Nusselt number $Nu/Ra^{1/2}$ as a function of axial coordinate x for various values of the magnetic field parameter $M = 0.0.$, 1.0, and 2.0, where $a = 0.0$, 0.2 and $\Omega = 10^0$. It is observed that this quantity varies periodically in the direction of x when $a \neq 0$ (wavy surface), also one can see that increasing the magnetic field M tends to decrease the amplitude of the local Nusselt number (decrease the heat transfer rate as compared with the limiting case of a smooth cone). Again, the range of x in the figure is from 0 to 4 which corresponds

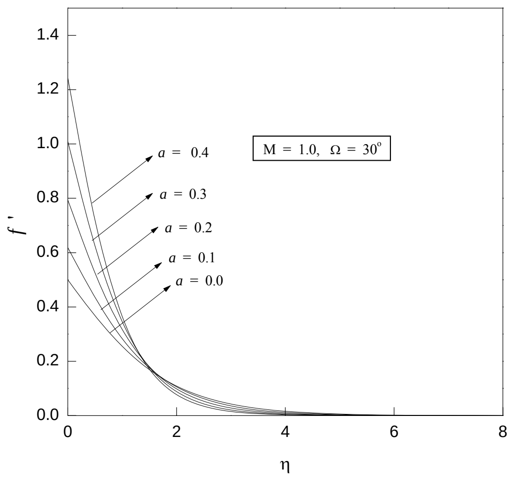

Fig. 5. Effect of a on the velocity against η, $x = 1.0$

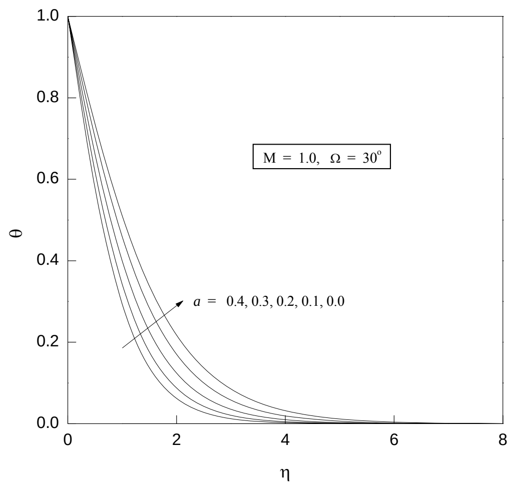

Fig. 6. Effect of a on the temperature against η, $x = 1.0$

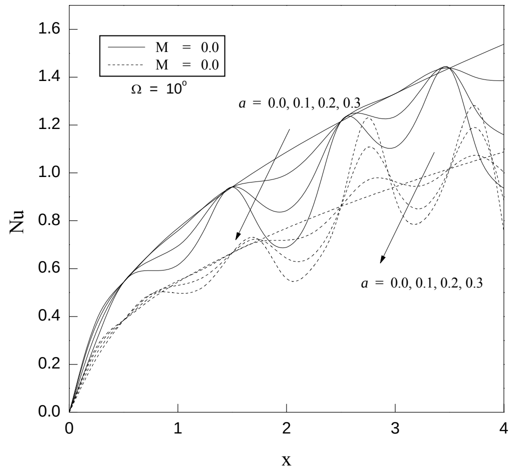

Fig. 7. Effect of a on the local Nusselt number against x

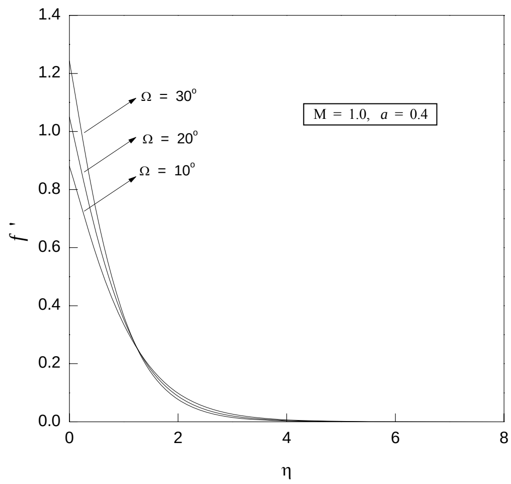

Fig. 8. Effect of Ω on the velocity against η, $x = 1.0$

to two complete cycles of the undulations as shown in Fig. (1). The raise and fall of the curves is seen to follow the change of the surface contour.

Figures (5) and (6) illustrate the effect of wave amplitude on velocity and temperature profiles, it is clear that as the amplitude-wavelength a, increases the velocity increases while the temperature decreases. Fig. (7) shows the streamwise distribution of the local Nusselt number $Nu/Ra^{1/2}$ for various values of amplitude-–wavelength ratio $a = 0.0$, 0.1, 0.2 and 0.3, with $\Omega = 10^0$ and $M = 1.0$. Increasing the amplitude–wavelength ratio a leads to a greater fluctuation of the local Nusselt number with the streamwise coordinate x. Moreover, the increase of the amplitude–wavelength ratio, on the average, tends to decrease the local heat transfer rate as compared with the limiting case of a smooth cone. In addition, the harmonic curves for the local Nusselt number as functions of streamwise coordinate have a frequency twice the frequency of the wavy conical surface.

Figures (8) and (9) display the effect of the half angle of the smooth conical Ω, with increasing Ω tends to increase the velocity profile and decrease the temperature profiles. Fig. (10) illustrates the effect of the half cone angle $\Omega = 10^0$, 20^0 and 30^0 where $a = 0.0,\ 0.2$ and $M = 1.0$ on the rate of heat transfer in terms of local Nusselt number. As expected, the effects are more pronounced for larger full cone angles.

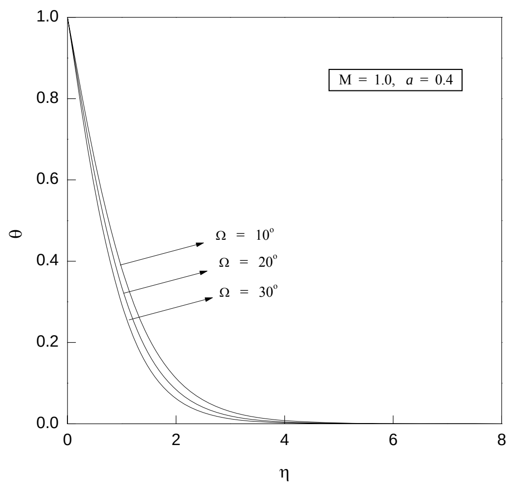

Fig. 9. Effect of Ω on the temperature against η, $x = 1.0$

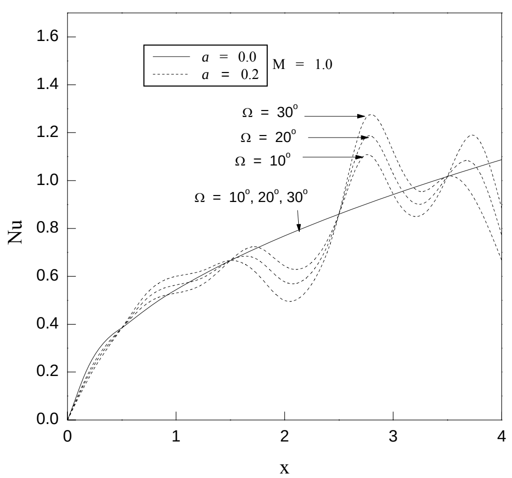

Fig. 10. Effect of Ω on the local Nusselt number against x

IV. CONCLUSION

The problem of steady-state, laminar heat transfer by natural convection boundary layer flow around a permeable wavy cone in the presence of magnetic field effect was considered. A set of non-similar governing differential equations was obtained and solved numerically by Runge-Kutta method. A representative set of numerical results for the velocity, temperature, as well as the local Nusselt number was presented graphically and discussed. It was found that the magnetic field retards the heat transfer process by decreasing the local Nusselt number and increasing the fluid temperature. In addition, the velocity field was strongly affected by the presence of the magnetic field. Increasing the magnetic field parameter leads to a greater fluctuation of the local Nusselt number with the streamwise coordinate x.

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Received: August 2, 2007.
Accepted: August 27, 2007.
Recommended by Subject Editor: Eduardo Dvorkin