HEAT AND MASS TRANSFER IN ELASTICO-VISCOUS FLUID PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE WITH HALL EFFECT

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***Abstract*— An unsteady hydromagnetic free convection flow of elastico-viscous fluid past an infinite vertical plate is investigated when the temperature and concentration are assumed to be oscillating with time and also the Hall effects are taken into account. Assuming constant suction at the plate, closed form solutions have been obtained for velocity, temperature and concentration distributions and presented graphically, for various values of the elastic parameter (α), Schmidt number (Sc), Magnetic parameter (M) and Hall parameter (m).**

***Keywords*— Hall effect, elastico-viscous, Heat-mass transfer.**

I. INTRODUCTION

The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in science and technology. Such phenomenon is observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth and so on. In nature and industrial applications many transport processes exist where the heat and mass transfer takes place simultaneously as a result of combined effects of thermal diffusion and diffusion of chemical species. In addition, the phenomenon of heat mass transfer is also encountered in chemical processes industries such as food processing and polymer production. Soundalgekar and Warve (1977) have analyzed two dimensional unsteady free convection flows, past an infinite vertical plate with oscillating wall temperature and constant suction. The effects of suction, which oscillates in time about a constant mean, have also been studied by them. An extensive contribution on heat and mass transfer flows has been made by Khair and Bejan (1985). Lin and Wu (1995) have analyzed the problem of simultaneous heat and mass transfer with entire range of buoyancy ratio for most practical chemical species in dilute and aqueous solutions. Muthukumarswamy *et al.* (2001) studied the heat and mass transfer effects on flow past an impulsively started infinite vertical plate. The solution was derived using the Laplace transform technique, and the effects of Grashof number, Prandtl number and Schmidt number were discussed.

Magnetohydrodynamics (MHD) is currently undergoing a period of great enlargement and differentiation of subject matter. The interest in these new problems originates from their importance in liquid metals, electrolytes and ionized gases. The MHD heat and mass transfer processes are of interest in power engineering, metallurgy, astrophysics, and geophysics. *Singh et al.* (2003) studied the MHD heat and mass transfer in a flow of viscous incompressible fluid past an infinite vertical plate. Assuming oscillatory suction normal to the plate, they solved the problem analytically. The problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to vertical surface was analyzed by Chen (2004). The strength of the magnetic field was found to have an appreciable effect on skin friction coefficient, the Nusselt number and the Sherwood number. In the above investigations, the effects of Hall current are not considered. Therefore, the results in these investigations cannot be applied to the flow of ionized gases. This is because in an ionized gas where the density is low and (or) the applied magnetic field is strong, the effect of Hall current may be significant. Katagiri (1969) has studied the effects of Hall current on the magnetohydrodynamic boundary layer flow past a semi-infinite plate. Pop and Soundalgekar (1974) and Gupta (1975) have investigated the effects of Hall current on the steady hydromagnetic flow in an incompressible viscous fluid. Hossain and Rashid (1987) discussed the effects of Hall current on unsteady free convection flow along a porous plate in the presence of foreign gases such as H_2, CO_2, and NH_3 subjected to a transpiration velocity inversely proportional to square root of time. Pop and Wattanable (1994) considered the Hall effect on magneto-hydrodynamic free convection about a semi infinite vertical plate and solved the problem numerically by employing difference-differential method in combination with Simpson's rule. Acharya *et al.* (1995, 2001) analyzed Hall effect with simultaneous thermal and mass diffusion on unsteady hydromagnetic flow past an vertical plate. Assuming constant suction/injection normal to the plate, they solved the problem analytically. The results are discussed with respect to hydromagnetic parameter, Hall parameter, suction parameter, and Schmidt number. Aboeldahab and Elbarbary (2001) discussed heat and mass transfer along a vertical plate under the combined buoyancy force effects of thermal and species diffusion in the presence of transversely applied magnetic field and taking Hall effect into account. The system of non-linear equations is solved by using Runge-Kutta methods. Recently Sharma and Chaudhary (2005) studied the MHD heat and mass transfer along a vertical plate immersed in porous medium taking Hall effect into account. The effects of Hall parameter, magnetic field parameter, Schmidt number, Prandtl number were discussed for two cases, when *G*>0 and when *G*<0.

Despite the above studies, attention has hardly been focused to study the effects of the Hall current on unsteady hydromagnetic non-Newtonian fluid flows. Such work seems to be important and useful partly for gaining a basic understanding of such flows, and partly for possible applications of these fluids in chemical process industries, food and construction engineering, movement of biological fluids, in petroleum and production and in power engineering. Another important field of application is the electromagnetic propulsion. The study of such systems, which is closely associated with magneto-chemistry, requires a complete understanding of the equation of state, shear stress-shear rate relationship, thermal conductivity, electric conductivity and radiation. Some of these properties will undoubtedly be influenced by the presence of an external magnetic field. Sarpkaya (1961) discussed the steady flow of a uniformly conducting non-Newtonian incompressible fluid between two parallel plates. The fluid considered is under the influence of constant pressure gradient.

In the present analysis, it is proposed to study the effect of simultaneous heat and mass transfer on the flow of elastico-viscous fluid past an impulsively started infinite vertical plate with mass transfer and taking Hall effect into account. Closed form analytical solutions have been obtained for the velocity, temperature and concentration distributions and are shown graphically.

II. MATHEMATICAL FORMULATION

The constitutive equations for the rheological equation of state for an elastico-viscous fluid (Walter's liquid B') are

$$p_{ik} = -p\,g_{ik} + p'_{ik} \tag{1}$$

$$p'_{ik} = 2\int_{-\infty}^{t} \psi(t-t')\, e^{(1)}_{ik}(t')\,dt' \tag{2}$$

in which

$$\psi(t-t') = \int_0^{\infty} \frac{N(\tau)}{\tau} e^{-(t-t')\tau}\, d\tau\,, \tag{3}$$

$N(\tau)$ is the distribution function of relaxation times. In the above equation p_{ik} the stress tensor, p an arbitrary isotropic pressure, g_{ik} is the metric tensor of a fixed coordinate system x_i and $e^{(1)}_{ik}$, the rate of strain tensor. It was shown by Walters (1964) that Eq.(2) can be put in the following generalized form which is valid for all types of motion and stress

$$p'^{ik}(x,t) = 2\int_{-\infty}^{t} \psi(t-t') \frac{\partial x^i}{\partial x'^m} \frac{\partial x^k}{\partial x'^r} e^{(1)mr}(x'\,t')\,dt', \tag{4}$$

where x'^i is the position at time t' of the element that is instantaneously at the point x^i at time "t". The fluid with Eq. of state (1) and (4) has been designated as liquid B'. In the case of liquids with short memories, *i.e.* short relaxation times, the above equation of state can be written in the following simplified form

$$p'^{ik}(x,t) = 2\eta_0\, e^{(1)ik} - 2k_0 \frac{\partial e^{(1)ik}}{\partial t}\,, \tag{5}$$

in which $\eta_0 = \int_0^{\infty} N(\tau)\,d\tau$ is limiting viscosity at small rates of shear, $k_0 = \int_0^{\infty} \tau N(\tau)\,d\tau$ and $\frac{\partial}{\partial t}$ denotes the convected time derivative.

We consider the unsteady flow of a viscous incompressible and electrically conducting elastico-viscous fluid with oscillating temperature and concentration. The flow occurs along the x-axis which is taken to be along the plate and y-axis is taken normal to it. The plate starts moving in its own plane with velocity U_0 (a constant velocity). A uniform magnetic field is applied normal to the plate with constant suction as shown in Fig. 1. The equations governing the flow of fluid as follows

Equation of Continuity

$$\nabla.V = 0, \tag{6}$$

Momentum Equation

$$\frac{\partial V}{\partial t} + (V.\nabla)V = -\frac{1}{\rho}\nabla P + \nabla.p_{ij} + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) + \frac{1}{\rho}(J \times B), \tag{7}$$

Generalized Ohm's Law

$$J = \sigma(E + V \times B) - \frac{\sigma}{en_e}(J \times B - \nabla p_e), \tag{8}$$

where V=(u,v,w) is the velocity field, P is the pressure field, g is acceleration due to gravity, β the volumetric coefficient of the thermal expansion, β^* the volumetric coefficient of expansion with concentration, ρ the density of the fluid, J is the current density, B is the magnetic field, E is the electric field, p_{ij} is stress tensor. The effect of Hall current induces a force which causes cross flow in the z direction. Therefore the flow becomes three dimensional. It is assumed that there is no applied or polarization voltage so that E=0 and the induced magnetic field is negligible so that the total magnetic field $B=(0,B_0,0)$, where B_0 is the applied magnetic field parallel to y-axis. This assumption is justified when the magnetic Reynolds number (the ratio of the moduli of the convection term and diffusive term a non-dimensional number, strictly analogous in the properties and uses to the Reynolds number) is very small.

The generalized Ohm's law including Hall current is given in the form

$$\vec{J} + \frac{\omega_e \tau_e}{B_0}(\vec{J} \times \vec{B}) = \sigma\left(\vec{J} \times \vec{B} + \frac{1}{e n_e}\nabla p_e + \vec{E}\right), \tag{9}$$

where ω_e is the electron frequency, τ_e is the electron collision time, σ is the electrical conductivity, e is the electron charge, p_e is the electron pressure and n_e is the number density of electron. For weakly ionized gases the thermoelectric pressure and ion slip are considered negligible. Then equation (9) reduces to

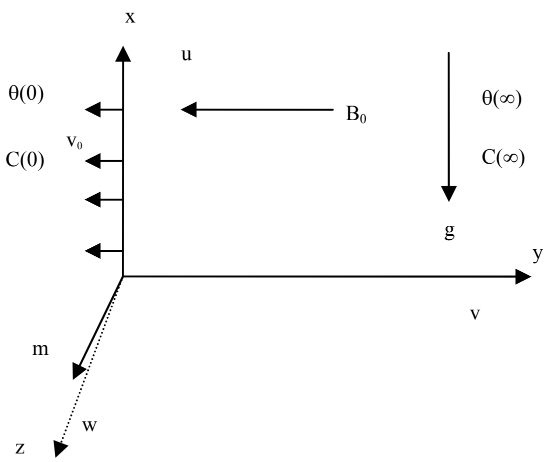

Figure 1: Physical model of the problem.

$$J_x = \frac{\sigma B_0}{1+m^2}(mu - w) \quad (10)$$

$$J_z = \frac{\sigma B_0}{1+m^2}(u + mw), \quad (11)$$

where u, v and w are x, y and z component of velocity vector respectively, m is the Hall parameter defined by $m = \omega_e \tau_e$.

As the plate is of infinite length, all the physical variables in this problem are functions of y and t only. Under this condition, Boussinesq approximation equations governing the flows are as follows

Equation of Continuity

$$\frac{\partial v}{\partial y} = 0 \quad (12)$$

$\Rightarrow v = -v_0$ where v_0 is constant suction velocity.

Momentum Equation

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - k_0\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\sigma B_0^2 (u + m w)}{\rho(1+m^2)}$$
$$+ g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \quad (13)$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} = \nu\frac{\partial^2 w}{\partial y^2} - k_0\frac{\partial^3 w}{\partial y^2 \partial t} - \frac{\sigma B_0^2 (w - mu)}{\rho(1+m^2)} \quad (14)$$

Energy Equation

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \quad (15)$$

Concentration Equation

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \quad (16)$$

where ρ is the density of the fluid, ν is the kinematic viscosity, k_0 the elastic parameter, T is the temperature, C is the concentration, κ the thermal conductivity, c_p is the specific heat of the fluid, D the chemical molecular diffusivity and g is the acceleration due to gravity. In Eq. (15) the terms due to viscous dissipation are neglected and in Eq. (16) the term due to chemical reaction is assumed to be absent.

The initial boundary conditions are

$$t \le 0,\ u(y,t) = w(y,t) = 0,\ T = 0,\ C = 0 \ \text{ for all } y$$

$$t > 0 \begin{cases} u(0,t) = U_0,\ w(0,t) = 0,\ T = T_\infty + e^{i\omega t}(T_w - T_\infty) \\ C(0,t) = C_\infty + e^{i\omega t}(C_w - C_\infty),\ \text{at } y = 0 \\ u(\infty,t) = w(\infty,t),\ T(\infty,t) = C(\infty,t) = 0 \ \text{ as } y \to \infty \end{cases} \quad (17)$$

where ω is frequency of oscillation, and subscript w and ∞ denotes the physical quantities at plate and in the free stream respectively.

We introduce the following non-dimensional parameters as follows

$$\eta = \frac{v_0 y}{\nu},\ \bar{t} = \frac{v_0^2 t}{4\nu},\ \bar{u} = \frac{u}{U_0},\ \bar{w} = \frac{w}{U_0}$$

$$\bar{\theta} = \frac{T - T_\infty}{T_w - T_\infty},\ \bar{C} = \frac{C - C_\infty}{C_w - C_\infty},$$

$$G = \frac{4 g\beta\nu(T_w - T_\infty)}{v_0^2 U_0} \ \text{(Grashof number)},$$

$$Gc = \frac{4 g\beta^*\nu(C_w - C_\infty)}{v_0^2 U_0} \ \text{(modified Grashof number)},$$

$$M = \frac{4B_0^2\sigma\nu}{\rho v_0^2 U_0} \ \text{(Hartman number)},$$

$$Pr = \frac{\nu\rho c_p}{\kappa} \ \text{(Prandtl number)},$$

$$\alpha = \frac{k_0 v_0^2}{\nu^2} \ \text{(elastic parameter)},$$

$$Sc = \frac{\nu}{D} \ \text{(Schmidt number)},$$

$$\Omega = \frac{4\nu\omega}{v_0^2} \ \text{(Non-dimensional frequency of oscillation)} \quad (18)$$

Substituting Eq. (19) in (14)–(17) and (18) and dropping the bars we get

$$\frac{\partial u}{\partial t} - \frac{4\partial u}{\partial \eta} = \frac{4\partial^2 u}{\partial \eta^2} - \alpha\frac{\partial^2 u}{\partial \eta^2 \partial t}$$
$$- \frac{M}{1+m^2}(mw + u) + G\theta + Gc\,C \quad (19)$$

$$\frac{\partial w}{\partial t} - \frac{4\partial w}{\partial \eta} = \frac{4\partial^2 w}{\partial \eta^2} - \alpha\frac{\partial^2 w}{\partial \eta^2 \partial t} - \frac{M}{1+m^2}(w - mu) \quad (20)$$

$$\frac{\partial \theta}{\partial t} - \frac{4\partial \theta}{\partial \eta} = \frac{4}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} \quad (21)$$

$$\frac{\partial C}{\partial t} - \frac{4\partial C}{\partial \eta} = \frac{4}{Sc}\frac{\partial^2 C}{\partial \eta^2} \quad (22)$$

and the boundary conditions for Eq. (19)–(22) are

$$t \le 0,\ u(\eta,t) = w(\eta,t) = \theta(\eta,t) = C(\eta,t) = 0 \ \forall\ \eta$$

$$t > 0 \begin{cases} u(0,t) = 1,\ w(0,t) = 0,\ \theta(0,t) = e^{i\Omega t}, \\ C(0,t) = e^{i\Omega t} \ \text{at } \eta = 0 \\ u(\infty,t) = w(\infty,t) = 0, \\ \theta(\infty,t) = C(\infty,t) = 0 \ \text{as } \eta \to \infty \end{cases} \quad (23)$$

III. SOLUTION

The Eqs. (19) and (20) can be combined using the complex variable

$$\psi = u + iw \quad \cdot \tag{24}$$

Equations (19) – (20) give

$$\frac{\partial^2 \psi}{\partial \eta^2} - \alpha \frac{\partial^3 \psi}{\partial \eta^2 \partial t} - \frac{1}{4}\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial \eta} - \frac{M(1-im)}{4(1+m^2)}\psi = -\frac{G\theta}{4} - \frac{Gc\,C}{4} \cdot \tag{25}$$

Using (24), we get boundary conditions as

$$\begin{aligned} &\psi(0,t) = 1,\ \psi(\infty,t) = 0,\ C(0,t) = e^{i\Omega t} \\ &\theta(0,t) = e^{i\Omega t},\ \theta(\infty,t) = 0,\ C(\infty,t) = 0 \end{aligned} \tag{26}$$

Putting $\theta(\eta, t) = e^{i\Omega t} f(\eta)$ in Eq. (21), we get

$$f''(\eta) + \Pr f'(\eta) - \frac{i\Omega \Pr}{4} f(\eta) = 0 \ , \tag{27}$$

which has to be solved under the boundary conditions

$$f(0) = 1, \qquad f(\infty) = 0 \qquad \ldots(28)$$

Hence $f(\eta) = e^{1/2[-\Pr - \sqrt{\Pr^2 + i\Omega \Pr}\,]\eta}$

$$\Rightarrow \theta(\eta, t) = e^{i\Omega t - \frac{\eta}{2}[\Pr + \sqrt{\Pr^2 + i\Omega\Pr}\,]}$$

Separating real and imaginary part, the real part is given by

$$\theta_r(\eta, t) = \cos\left\{\Omega t - \frac{\eta}{2} R_1 \sin\frac{\beta_1}{2}\right\} e^{-\frac{\eta}{2}(\Pr + R_1\cos\frac{\beta_1}{2})}, \tag{29}$$

where

$$R_1 = \Pr^{1/2}(\Pr^2 + \Omega^2)^{1/4} \ ,\beta_1 = \tan^{-1}\left(\frac{\Omega}{\Pr}\right) \tag{30}$$

Putting $C(\eta, t) = e^{i\Omega t} g(\eta)$ in Eq. (23), we get

$$g''(\eta) + Sc\, g'(\eta) - \frac{i\Omega\, Sc}{4} g(\eta) = 0 \tag{31}$$

which can be solved under the boundary conditions

$$g(0)=1,\ g(\infty)=0.$$

Hence $g(\eta) = e^{1/2[-Sc - \sqrt{Sc^2 + i\Omega Sc}\,]\eta}$

$$\Rightarrow C(\eta, t) = e^{i\Omega t - [Sc + \sqrt{Sc^2 + i\Omega Sc}\,]\frac{\eta}{2}} . \tag{32}$$

Separating real and imaginary part, the real part is given by

$$C_r(\eta, t) = \cos\left\{\Omega t - \frac{\eta}{2} R_2 \sin\frac{\beta_2}{2}\right\} e^{-\frac{\eta}{2}(Sc + R_2\cos\frac{\beta_2}{2})}, \tag{33}$$

where

$$\left.\begin{aligned} R_2 &= Sc^{1/2}(Sc^2 + \Omega^2)^{1/4} \\ \beta_2 &= \tan^{-1}\left(\frac{\Omega}{Sc}\right) \end{aligned}\right\} .$$

In order to solve

$$\frac{\partial^2 \psi}{\partial \eta^2} - \alpha \frac{\partial^3 \psi}{\partial \eta^2 \partial t} - \frac{1}{4}\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial \eta} - \frac{M(1-im)}{4(1+m^2)}\psi = -\frac{G\theta}{4} - \frac{GcC}{4}$$

substituting $\psi = e^{i\Omega} F(\eta)$ and using boundary conditions

$$\left.\begin{aligned} F(0) &= e^{-i\Omega t} \\ F(\infty) &= 0 \end{aligned}\right\} . \tag{34}$$

Separating real and imaginary part, we get

$$\begin{aligned} u = {} & e^{-\eta a_4}[\{\cos\eta a_5 + (A_9A_7 + A_{10}A_{12})\cos(\Omega t - \eta a_5)\} \\ & + \{(A_8A_9 + A_{11}A_{12})\sin(\Omega t - \eta a_5)\}] \\ & - e^{\eta a_6}\left[A_9A_7\cos(\Omega t - \eta a_7) + A_9A_8\sin(\Omega t - \eta a_7)\right] \\ & - e^{-\eta a_8}\left[A_{10}A_{12}\cos(\Omega t - \eta a_9) + A_{11}A_{12}\sin(\Omega t - \eta a_9)\right], \end{aligned} \tag{35}$$

$$\begin{aligned} w = {} & e^{-\eta a_4}[\{\sin\eta a_5 + (A_9A_7 + A_{10}A_{12})\sin(\eta t - \eta a_5\} \\ & - \{(A_8A_9 + A_{11} + A_{12})\cos(\Omega t - \eta a_5)\}] \\ & - e^{-\eta a_6}[A_9A_7\sin(\Omega t - \eta a_7) - A_9A_8\cos(\Omega t - \eta a_7)] \\ & - e^{-\eta a_8}[A_{10}A_{12}\sin(\Omega t - \eta a_9) - A_{11}A_{12}\cos(\Omega t - \eta a_9)] \end{aligned} \tag{36}$$

where $a_1 = \frac{\alpha}{4}$, $a_2 = \frac{M}{4(1+m^2)}$, $a_3 = \frac{\Omega}{4} - \frac{Mm}{4(1+m^2)}$,

$A_1 = 4(a_2 + a_1a_3)$, $A_2 = 4(a_3 - a_1a_2)$, $A_3 = 1 + r^{1/4}\cos\gamma/2$,

$A_4 = r^{1/4}\sin\gamma/2$, $r = (1 + A_1^2)^2 + A_2^2$, $\gamma = \tan^{-1}\frac{A_2}{1+4A_1}$,

$A_4 = \frac{(A_3 - A_4)}{2(1+a_1^2)}$, $a_5 = \frac{(a_1A_3 + A_4)}{2(1+a_1^2)}$, $a_6 = \frac{1}{2}(\Pr + R_1\cos\frac{\beta_1}{2})$,

$a_8 = \frac{1}{2}(Sc + R_2\sin\beta_2/2)$, $a_7 = \frac{1}{2}R_1\sin\frac{\beta_1}{2}$, $a_9 = \frac{1}{2}(R_2\cos\beta_2/2)$,

$R_2 = Sc^{1/2}(Sc^2 + \Omega^2)^{1/4}$, $R_1 = \Pr^{1/2}(\Pr^2 + \Omega^2)^{1/4}$, $\alpha_1 = \tan^{-1}\left(\frac{\Omega}{4}\right)$,

$\beta_1 = \tan^{-1}\left(\frac{\Omega}{Sc}\right)$, $A_5 = a_6^2 - a_7^2 + 2a_1a_6a_7 - a_6 - a_2$,

$A_6 = 2a_6a_7 - a_1(a_6^2 - a_7^2) - a_7 - a_3$, $A_7 = \frac{G}{4(A_7^2 + A_8^2)}$,

$A_8 = a_8^2 - a_9^2 + 2a_1a_8a_9 - a_8 - a_2$, $A_{10} = \frac{Gc}{4(A_8^2 + A_9^2)}$ and

$A_9 = 2a_8a_9 - a_1(a_8^2 - a_9^2) - a_9 - a_3$.

IV. DISCUSSION

The effect of Hall current on MHD free convection flow of elastico-viscous fluid past an impulsively started infinite vertical plate with mass transfer has been carried out in preceding sections. In order to get physical insight into the problem, the velocity, temperature, concentration fields, shear stress, rate of heat and mass transfer have been discussed by assigning numerical value to M (magnetic parameter), m (Hall parameter), α (Non-Newtonian parameter), Sc (Schmidt number) and Ω (frequency parameter). The values of Prandtl number (Pr), are taken equal to 3 and 10 which represent the saturated liquid (Freon $cF_2\,c\ell_2$) at 272.3^0 K and Gasoline at 1 atm. Pressure and 20^0C respectively. The value of Grashof number (G) and modified Grashof number (Gc) are taken equal to 5 and 2 respectively.

Figure 2 depicts the variation of velocity component u, for G=5.0, Gc=2.0 and Pr=3 taking different values of m, M, Sc, Ω and α. It is observed that an increase in the Hall parameter ($m=\omega_e\tau_e$) leads to a rise in the velocity while reverse effect is observed for applied magnetic field (M) for both Newtonian and non-Newtonian fluid. The velocity is greater for Ammonia (Sc=0.78) than for Helium (Sc=0.30). An increase in frequency increases the velocity. It is also noticed that velocity for Newtonian fluid (α=0) is more than the non-Newtonian fluid ($\alpha\neq0$). Figure 3 represents the velocity component w, for G=5, Gc=2 and Pr=3. An increase in applied magnetic field (M), Schmidt number (Sc) and frequency (Ω) decreases the velocity in z-direction for both the fluids whereas increase in Hall parameter (m) increases the velocity. Further it is noticed that w-component of velocity of Newtonian fluid is lower than that of elastico-viscous fluid. The velocity profile (w) for Gr=5, Gc=2, M=5, m=0.5, Ω=1, Pr=7 and for α=0 is compared to the available results of Sharma and Chaudhary (2005). It is observed that agreement with the result obtained by Sharma and Chaudhary (2005) is excellent. Here it is concluded that the maximum velocity occurs in the vicinity of the plate and as $\eta\to\infty$ the velocity profiles terminate to zero. The temperature profiles have been shown in Fig. 4 for Pr=3 and 7. The temperature distribution increases with increase in frequency. When Pr=7, numerical results are same as that of Sharma and Chaudhary (2005). The maximum value occurs near the plate and then faded away from the plate. Figure 5 represents the concentration profiles for Helium (Sc=0.30) and Ammonia (Sc=0.78). We noticed that effect of increasing value of Sc is to decrease the concentration profiles. This is consistent with the fact that increase in Sc means decrease of molecular diffusivity D that results in decrease of concentration boundary layer. Hence concentration of the species is higher for small values of Sc and lower for larger values of Sc. Further it is observed that value of concentration increases at each point in the flow field with increase in Ω (frequency).

Knowing the velocity field it is important from a practical point of view to know the effect of physical parameters, Sc, M, m and α on skin friction. We now calculate the skin friction from these relations

$$\tau_{xw} = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

in non-dimensional form it takes

$$\tau_1 = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}$$

where τ_1 is the x-component of skin friction.

Similarly z-component of skin friction τ_2 is given as

$$\tau_2 = \left[\frac{\partial w}{\partial \eta}\right]_{\eta=0}$$

(in non-dimensional form).

The shearing stress along x-axis τ_1 is shown in Fig. 6 for different value of M, Sc and elastic parameter α It is observed that there is a rise in τ_1 with increasing Schmidt number but it falls with increasing M and α Figure 7 represents the shearing stress along z-axis (τ_2). From this we concluded that for non-Newtonian fluid τ_2 is greater than for Newtonian fluid. However, increase in M and Sc decreases the skin friction τ_2. Here it is also concluded that the values of τ_1 and τ_2 increase with the increase in Hall parameter (m).

The rate of heat transfer in terms of Nusselt number is given by

$$Nu = \frac{q\nu}{v_0\kappa(T_w - T_\infty)}$$

where $q = -\kappa \left.\frac{\partial T}{\partial y}\right|_{y=0}$ in non-dimensional form it is given by

$$Nu = -\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} = \frac{1}{2}\left[\Pr\cos\Omega t + R_1\cos\left(\Omega t + \frac{\beta_1}{2}\right)\right] \quad (37)$$

The rate of mass transfer is given by

$$J^*\,(\text{Diffusion flux}) = -\rho D \left.\frac{\partial C^*}{\partial y}\right|_{y=0}$$

The coefficient of mass transfer, which is generally known as Sherwood number S_h is given by

$$S_h = \frac{J^*\nu}{v_0\rho D(C_w - C_\infty)} = -\left.\frac{\partial C}{\partial\eta}\right|_{y=0}$$

$$= \frac{1}{2}\left[Sc\cos\Omega t + R_2\cos\left(\Omega t + \frac{\beta_2}{2}\right)\right] \quad (38)$$

Numerical values of heat and mass transfer rate are calculated from Eqs. (37) and (38) and these values are plotted in Fig. 8 and 9. Figure 8 gives the heat transfer for Pr=3 and Pr=10 against the frequency (Ω). It is noticed that an increase in Prandtl number decreases the rate Nu. The rate of mass transfer (S_h) for different values of Sc is shown in Fig. 9. It is concluded that Sherwood number decreases with increasing Schmidt number. There is a phase lead in heat and mass transfer.

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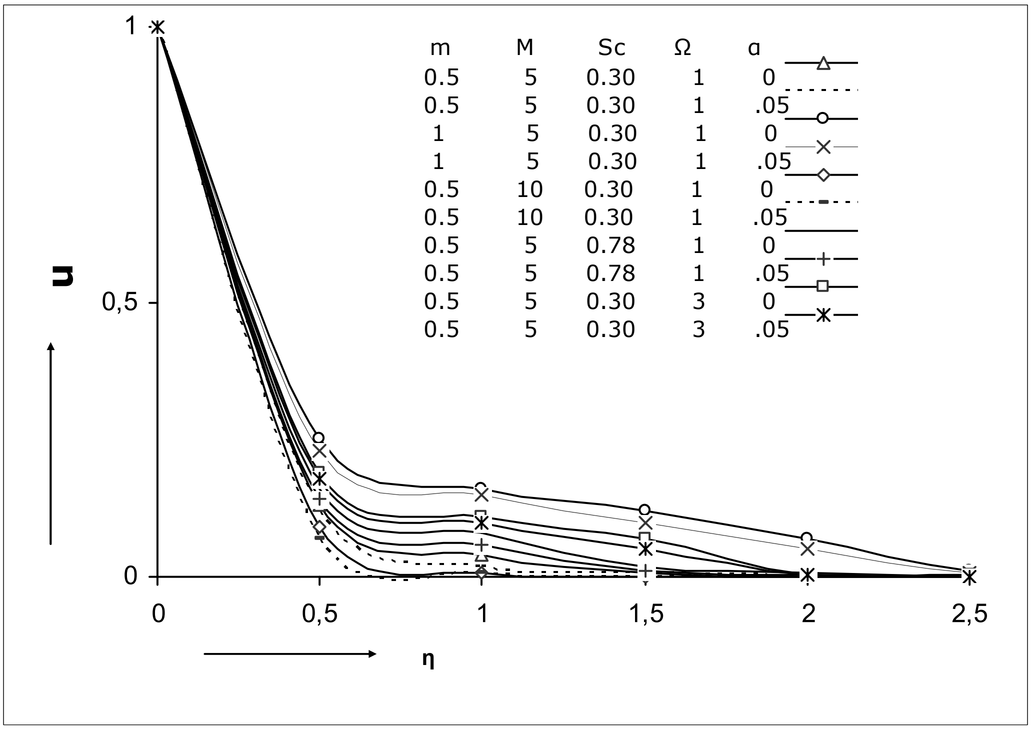

Figure 2. Variation of velocity component u against η, for G=5, Gc=2, Pr=3, $\Omega t = \pi / 2$

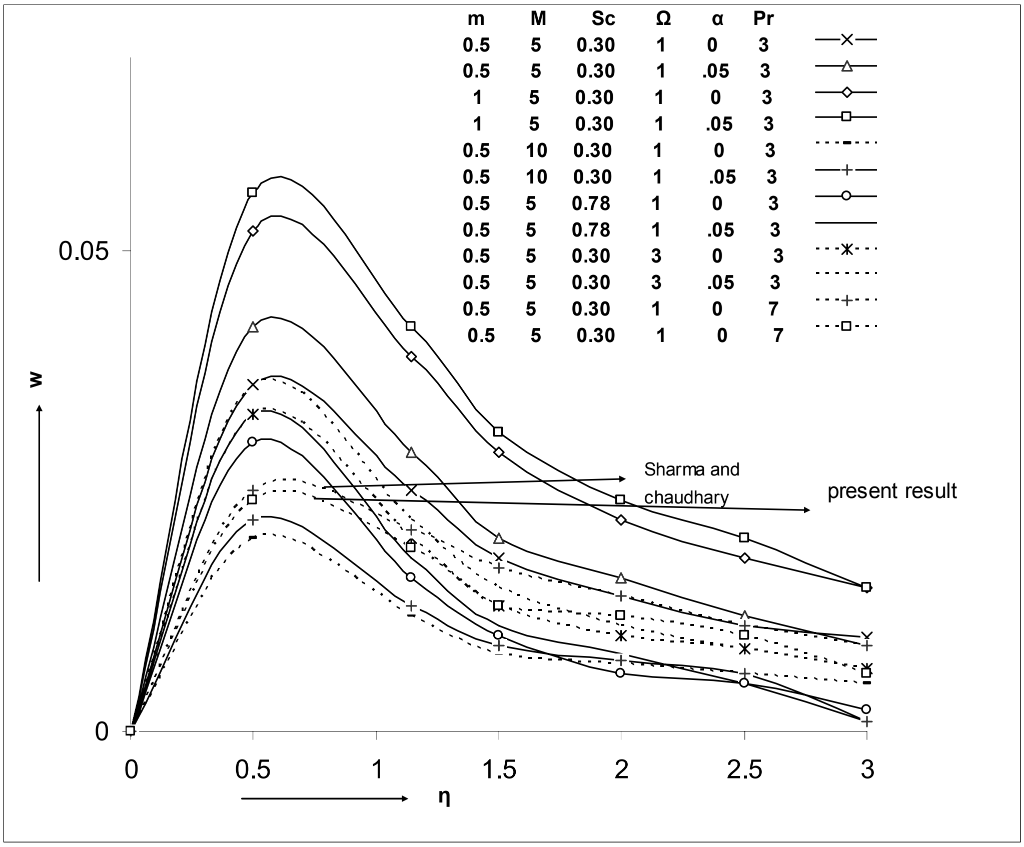

Figure 3: variation of velocity component w, against η, for G=5, Gc=2, $\Omega t = \pi / 2$

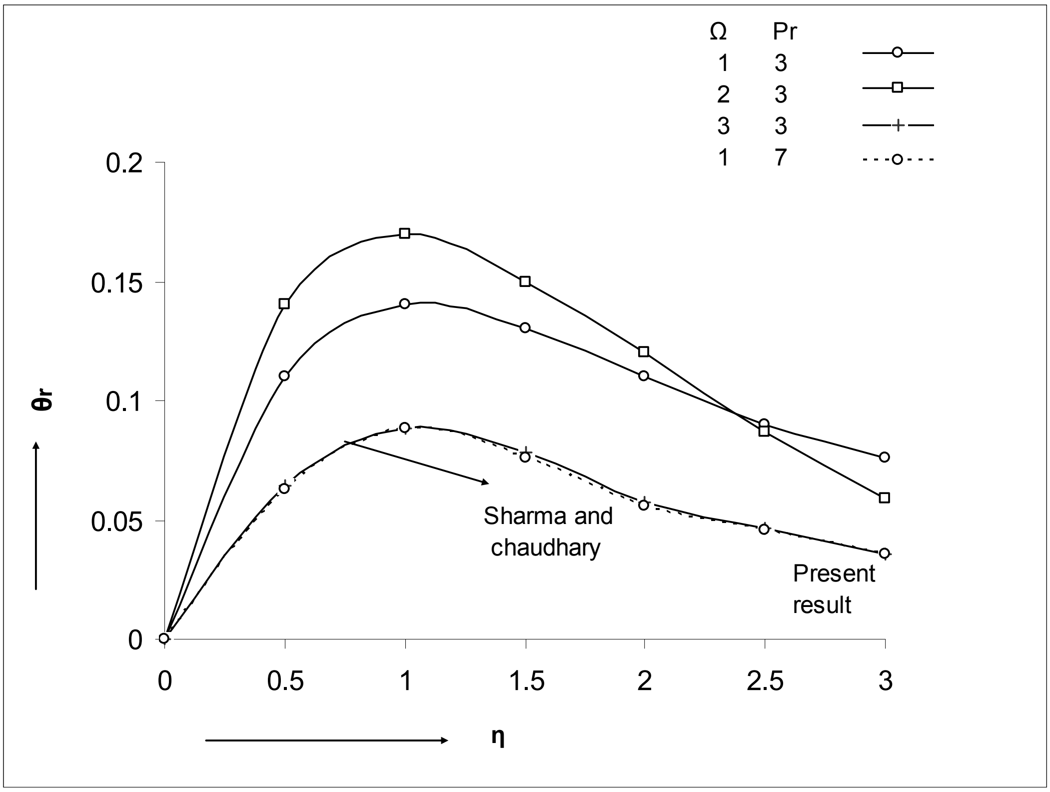

Figure 4: variation of temperature θr against η , $\Omega t = \pi / 2$

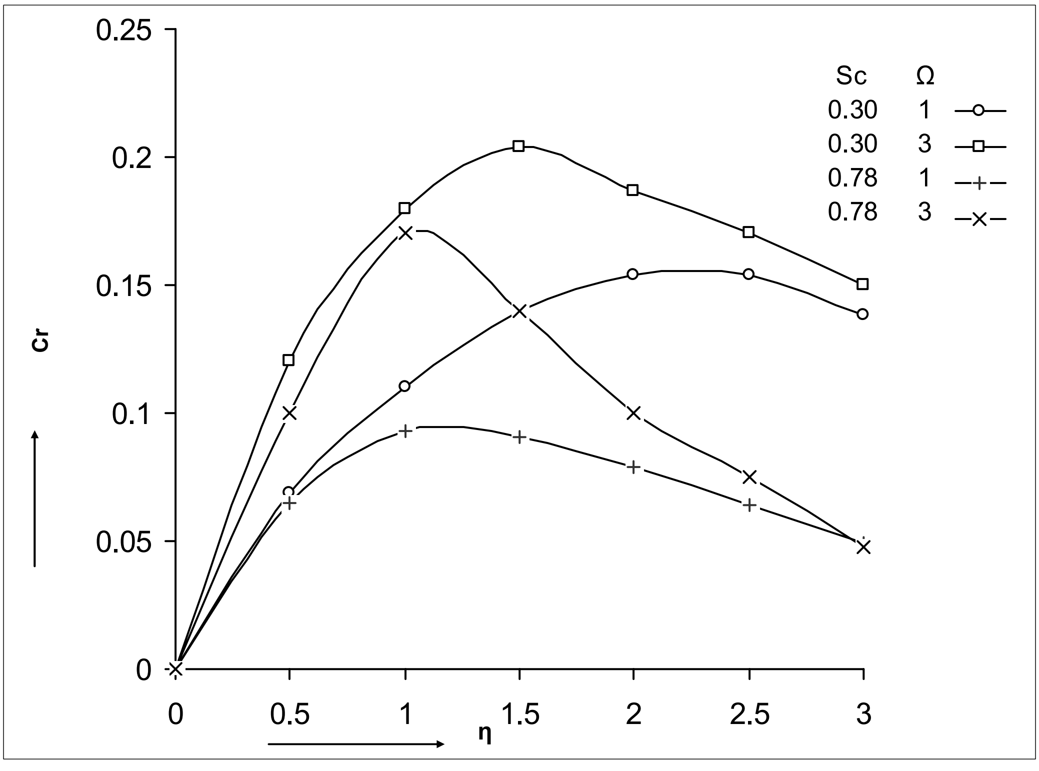

Figure 5: variation of concentration Cr against η, for $\Omega t = \pi / 2$

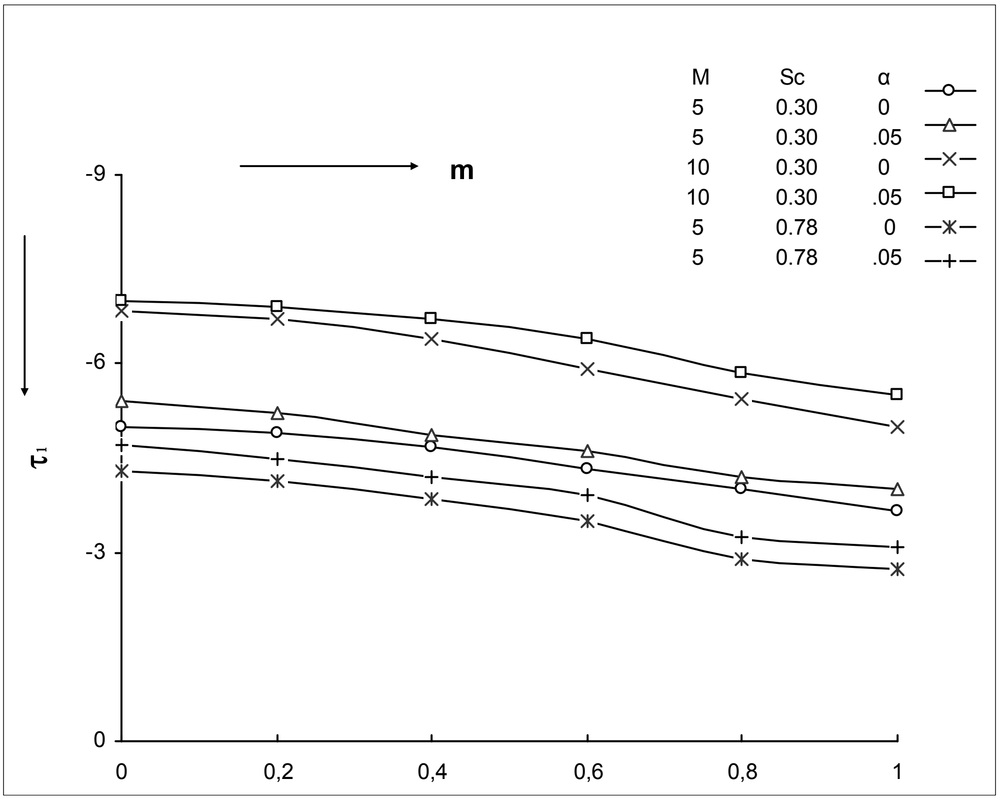

Figure 6: Variation of shearing stress τ_1 against m , for G=5,Gc=2,Pr=3, $\Omega = 1, \Omega t = \pi / 2$

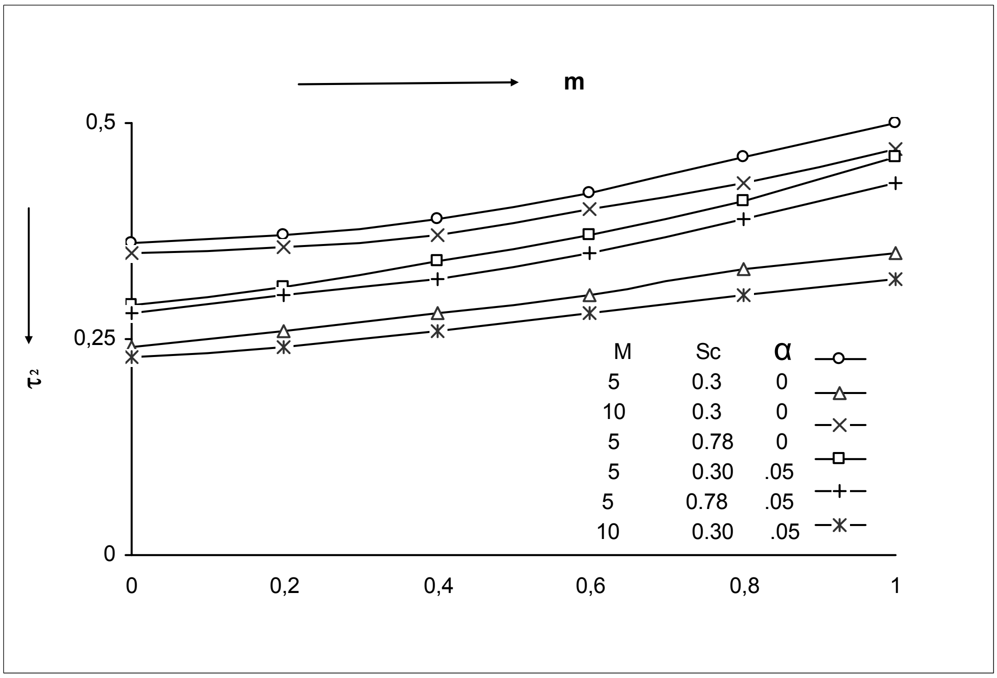

Figure 7: Variation of shearing stress τ_2 against m, for G=5,Gc=2,Pr=3, $\Omega = 1, \Omega t = \pi / 2$

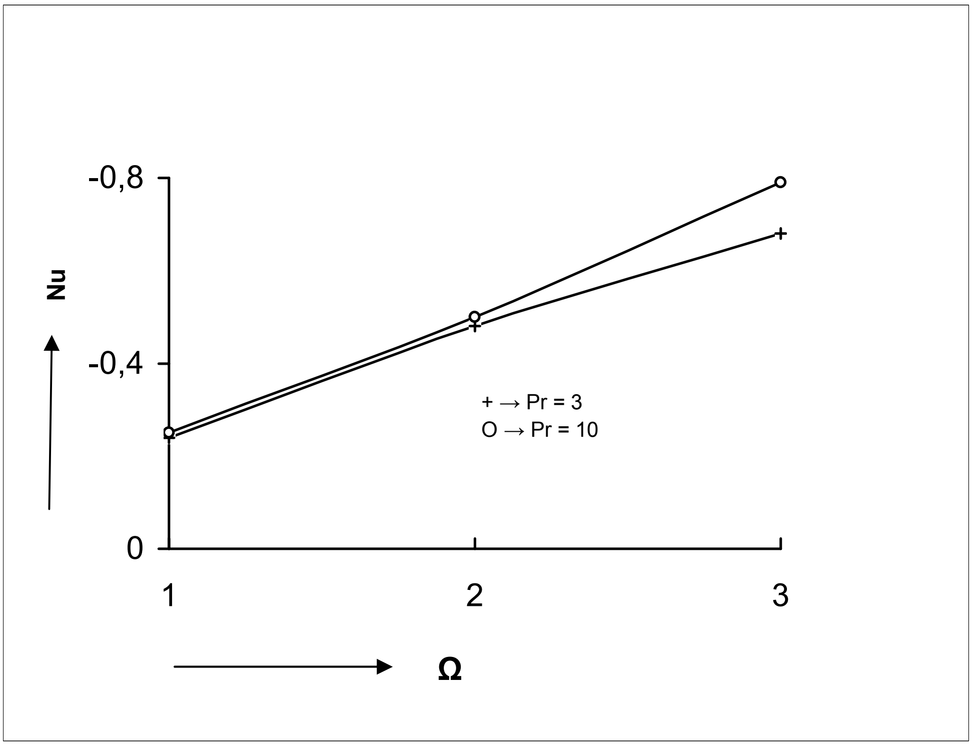

Figure 8: Rate of heat transfer for $\Omega t = \pi / 2$ against the frequency Ω

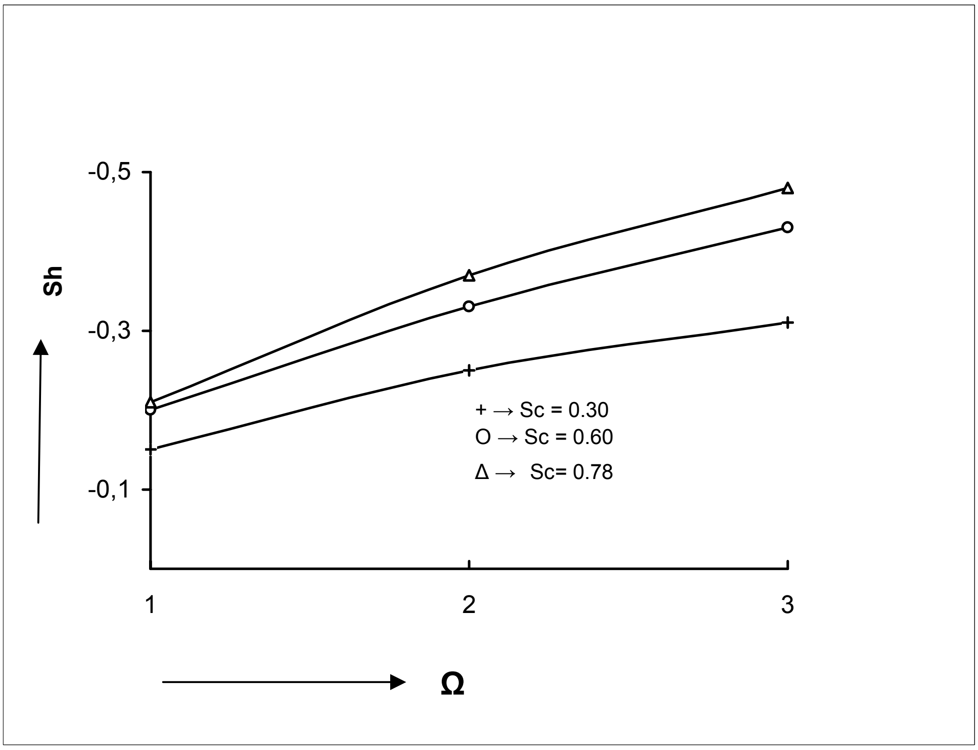

Figure 9: Rate of mass transfer for $\Omega t = \pi / 2$ against the frequency Ω

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