Latin American Applied Research 35:135-141 (2005)

DISCRETE EVENT CONTROL OF TIME–VARYING PLANTS

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***Abstract*— This paper studies the use of Quantized State Control (QSC) in Time–Varying (TV) plants. Making use of a Lyapunov analysis, the stability properties of Time Invariant QSC are extended to the non–stationary case. Then, based on the resulting stability theorem, a design algorithm is developed. Finally, the use of this algorithm –which allows the design of QSC controllers according to stability and convergence speed features– is shown with the design and the simulation of an illustrative example.**

***Keywords*— Digital Control, Nonlinear Control, Discrete Event Systems.**

I. INTRODUCTION

Quantized State Control (Kofman, 2003) is a methodology which allows the digital implementation of previously designed continuous controllers based on the quantization of their state variables. The resulting digital controller –which can be described by a discrete event system– is completed with an asynchronous sampling scheme (Sayiner *et al.*, 1993) so that, ignoring the temporal errors introduced by the clock of the digital device, no time discretization is performed.

The use of QSC instead of classic discrete time digital controllers permits to conserve the region of attraction in nonlinear systems improving also the dynamic response and reducing the computational costs as well as the information transmitted between controllers and plants. In the LTI case, digital QSC controllers also ensure that the resulting trayectories do not differ from the ones obtained with the ideal continuous controllers in more than a bound which can be calculated with a closed formula (Kofman, 2002).

Since QSC takes into account the presence of quantization, it does not guarantee asymptotic stability but *ultimately boundedness* of the solutions (Khalil, 1996). This fact, which seems to be a disadvantage, is common to all the quantization based approaches (Delchamps, 1990; Brockett and Liberzon, 2000; Elia and Mitter, 2001). Moreover, in any sampled data control system where the effects of A/D and D/A converters are taken into account the theory can only talk about ultimate bounds (Miller *et al.*, 1989; Hou *et al.*, 1997).

The original definitions of QSC and the mentioned properties were established under the assumption of a Time–Invariant (TI) plant with a TI controller. Although the restriction on the controller cannot be avoided[1], there is no reason to consider only TI plants.

Taking into account the last remark, this work attempts to extend the properties and design algorithms of TI QSC to the Time Varying case.

The paper is organized as follows:

After recalling the principles and properties of QSC (Section II), the main results of QSC with TV–plants –a stability Theorem and a design algorithm– are presented in Section III. Finally, these results are illustrated with the design and simulation of a simple example.

II. QUANTIZED STATE CONTROL

QSC is a digital control scheme in which a continuous plant is controlled by a discrete event system obtained with the quantization of a continuous controller.

A. Quantized State Systems

Consider the State Equation System (SES) given by:

$$\begin{cases} \dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= g(x(t), u(t)). \end{cases} \tag{1}$$

Related to this system, an associated Quantized State System (QSS) (Kofman and Junco, 2001) is given by:

$$\begin{cases} \dot{x}(t) &= f(q(t), u(t)) \\ y(t) &= g(q(t), u(t)). \end{cases} \tag{2}$$

In this system, $q(t)$ (quantized variables) and $x(t)$ (state variables) are related (componentwise) by hysteretic quantization functions, which are defined as follows:

Definition 1. *Hysteretic Quantization Function.*

Let $Q = \{Q_0, Q_1, ..., Q_r\}$ be a set of real numbers where $Q_{k-1} < Q_k$ with $1 \leq k \leq r$. Let Ω be the set of piecewise continuous real valued trajectories and let $x_i \in \Omega$ be a continuous trajectory. Let $b : \Omega \to \Omega$ be

[1] In the general case it is not possible to obtain a discrete event description of a quantized time–varying system.

a mapping and let $q_i = b(x_i)$ where the trajectory q_i satisfies:

$$q_i(t) = \begin{cases} Q_m & \text{if} \quad t = t_0 \\ Q_{k+1} & \text{if} \quad x_i(t) = Q_{k+1} \wedge \\ & \quad\quad \wedge q_i(t^-) = Q_k \wedge k < r \\ Q_{k-1} & \text{if} \quad x_i(t) = Q_k - \varepsilon \wedge \\ & \quad\quad \wedge q_i(t^-) = Q_k \wedge k > 0 \\ q_i(t^-) & \quad\quad \text{otherwise} \end{cases} \tag{3}$$

and

$$m = \begin{cases} 0 & \text{if } x_i(t_0) < Q_0 \\ r & \text{if } x_i(t_0) \geq Q_r \\ j & \text{if } Q_j \leq x(t_0) < Q_{j+1}. \end{cases}$$

Then, the map b is a hysteretic quantization function*.*

The discrete values Q_k are called *quantization levels* and the distance $Q_{k+1} - Q_k$ is defined as the *quantum*, which is usually constant. The width of the hysteresis window is ε and, as it was shown in Kofman *et al.* (2001), it should be taken equal to the quantum to improve the accuracy. The values Q_0 and Q_r are the lower and upper saturation bounds.

A fundamental property of a Quantization Function with hysteresis when $t \geq t_0$ and $Q_0 \leq x_i(t) \leq Q_r$ is given by the following inequality

$$|q_i(t) - x_i(t)| \leq \max_{1 \leq i \leq r}(Q_i - Q_{i-1}, \varepsilon). \tag{4}$$

The role of the quantization functions in (2) is to convert the state trajectories ($x(t)$) into piecewise constant ones ($q(t)$). In that way, provided that $u(t)$ is piecewise constant, it results that the derivatives $\dot{x}(t)$ are also piecewise constant and then the state trajectories are piecewise linear.

As a consequence of these properties of the trajectory forms, QSS can be exactly represented by discrete event models within the DEVS formalism framework (Zeigler *et al.*, 2000). The DEVS model related to a generic QSS and the proof of the mentioned properties can be found in (Kofman and Junco, 2001).

The possibility of representing QSS by DEVS models and the fact that DEVS models can be simulated in real time by digital devices[2] (Zeigler and Kim, 1993) suggested the use of QSS as digital controllers and the definition of QSC.

B. QSC Definition

Consider the Continuous Control System (CCS)

$$\begin{cases} \dot{x}_p(t) = f_p(x_p(t), u_p(t), t) \\ y_p(t) = g_p(x_p(t), t) \end{cases} \tag{5}$$

$$\begin{cases} \dot{x}_c(t) = f_c(x_c(t), u_c(t), u_r(t)) \\ y_c(t) = g_c(x_c(t), u_c(t), u_r(t)) \end{cases} \tag{6}$$

$$u_p(t) = y_c(t), \quad u_c(t) = y_p(t) \tag{7}$$

[2]DEVS representation of QSS is exact. However, real time simulation of DEVS has errors related to temporal resolution and round-off in digital devices

consisting of plant (5), controller (6) and their (ideal) interconnection (7).

It is being considered here that the plant could be time varying. When it comes to the controller, it is assumed that it is stationary and it has an input reference $u_r(t)$.

Definition 2. *Quantized State Controller.*

A QSS associated to a continuous controller (6) is called Quantized State Controller (QSC controller).

Definition 3. *QSC System.*

A QSC system is a control scheme composed by a continuous plant and a QSC controller connected through asynchronous A/D and D/A converters.

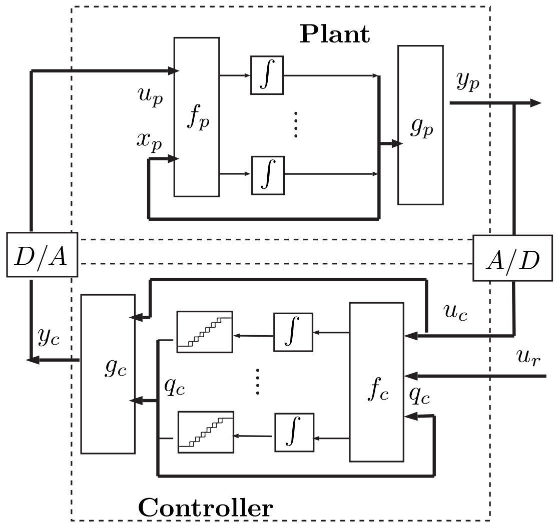

Figure 1. Block Diagram of the QSC system

Figure 1 shows a block diagram representation of a QSC system. Despite its state–equation–like representation, the controller is in fact a DEVS model.

The QSC implementation of the controller transforms (6) into the new set of equations:

$$\begin{cases} \dot{x}_c(t) = f_c(q_c(t), u_c(t), u_r(t)) \\ y_c(t) = g_c(q_c(t), u_c(t), u_r(t)). \end{cases} \tag{8}$$

The asynchronous sampling scheme (Sayiner *et al.*, 1993) implies that the A/D conversions are performed only when the analog input and the digital output of the converters differ in a quantity corresponding to one quantization interval. Then, they can be seen as quantization functions with hysteresis where the quantization intervals and the hysteresis windows have the same size. Similarly, D/A converters can be represented by quantization functions (but without hysteresis). Thus, the presence of the asynchronous converters transforms (7) into:

$$u_p(t) = y_{c_q}(t), \quad u_c(t) = y_{p_q}(t), \tag{9}$$

where variables $y_{c_q}(t)$ and $y_{p_q}(t)$ are quantized versions of the plant and the controller output variables

C. QSC Properties

The CCS closed loop equations can be derived from Eqs.(5)–(7) arriving to

$$\begin{cases} \dot{x}_p &= f_p(x_p, g_c(x_c, g_p(x_p,t), u_r), t) \\ \dot{x}_c &= f_c(x_c, g_p(x_p,t), u_r). \end{cases} \tag{10}$$

Let us define

$$\Delta x_c(t) = q_c(t) - x_c(t), \tag{11a}$$
$$\Delta y_p(t) = y_{p_q}(t) - y_p(t), \tag{11b}$$
$$\Delta y_c(t) = y_{c_q}(t) - y_c(t). \tag{11c}$$

Thus, from these definitions and Eqs.(5), (8) and (9), the QSC closed loop equations can be written as:

$$\begin{cases} \dot{x}_p &= f_p(x_p, g_c(x_c + \Delta x_c, g_p(x_p,t) + \\ & \qquad + \Delta y_p, u_r) + \Delta y_c, t) \\ \dot{x}_c &= f_c(x_c + \Delta x_c, g_p(x_p,t) + \Delta y_p, u_r). \end{cases} \tag{12}$$

Then, the QSC system (12) can be seen as a perturbed version of the original CCS (10).

From the property of the quantization functions – provided that the variables x_c, y_p and y_c do not reach the corresponding saturation bounds[3]– it also results that the perturbation terms are bounded by (4). Thus, the properties of the QSC implementation of a CCS can be studied by looking at the effects of bounded perturbations in the original closed loop system.

Based on this remark, the stability theorem of TI QSC (Kofman, 2003) and the stability and error bound estimation of LTI QSC (Kofman, 2002) were proven.

The stability theorem of TI QSC tells that, given a CCS in which the origin is an asymptotically stable equilibrium point, an appropriate quantization can be found so that the resulting QSC controller ensures ultimately boundedness of the solutions (for any given ultimate bound) conserving also the estimate region of attraction.

That appropriate quantization can be found according to desired ultimate bound following an algorithm which makes use of a Lyapunov function of the original CCS.

On the other hand, the theorem for LTI QSC tells that the QSC implementation of an asymptotically stable LTI CCS is always globally and ultimately bounded. The ultimate bound can be estimated with a closed formula. Moreover, the trajectories of the CCS and the resulting QSC never differ from each other in more than the mentioned bound.

Thus, the quantization choice which completes the QSC design can be made based on these properties, according to the desired ultimate bounds.

III. TIME VARYING QSC

The stability theorem and the corresponding design algorithm were based on a stationary Lyapunov analysis. This section introduces the main contributions of this work by extending those results to the time varying case.

[3]It will be considered that the non–saturation region is enough large so that the trajectories do not leave it.

A. Stability of TV QSC

When the reference trajectory $u_r(t)$ is zero (or constant), the QSC system (12) can be rewritten as

$$\dot{x} = f(x + \Delta x, \Delta y, t), \tag{13}$$

where $x \triangleq [x_p, x_c]^T$, $\Delta x \triangleq [0, \Delta x_c]^T$, $\Delta y \triangleq [\Delta y_p, \Delta y_c]^T$, and $f \triangleq [f_p, f_c]^T$.

With these definitions, the CCS (10) becomes

$$\dot{x} = f(x, 0, t) \triangleq \tilde{f}(x, t). \tag{14}$$

Then, the relationship between the stability properties of (14) and (13) can be stated in following theorem.

Theorem 1. *Let the origin be an asymptotically stable equilibrium point of the closed loop CCS* (14)*. Assume that function f is uniformly continuous and a uniformly continuously differentiable Lyapunov function $V(x,t)$ is known with*

$$W_1(x) \le V(x,t) \le W_2(x) \tag{15}$$

$$\frac{\partial V}{\partial x} \cdot \tilde{f}(x,t) + \frac{\partial V}{\partial t} \le -W_3(x) \tag{16}$$

$\forall t \ge 0$, $\forall x \in D$ being D a compact set which contains the origin and W_i are continuous positive definite functions in D.

Let $\Omega_{2_a} = \{x | W_2(x) \le a\}$ with a being an arbitrary positive constant which is enough small so that $\{x | W_1(x) \le a\}$ is a closed region inside D.

Let $\Omega_{1_b} = \{x | W_1(x) \le b\}$ being b an arbitrary positive constant ($b < a$) enough small so that $\Omega_{1_b} \subset \Omega_{2_a}$.

Then, a quantization can be found so that the QSC system trajectories starting in Ω_{2_a} finish inside Ω_{1_b}, reaching this region in finite time.

Proof. The derivative of $V(x,t)$ along the solutions of the QSC system (13) is

$$\begin{aligned} \dot{V}(x,t) &= \frac{\partial V}{\partial x} \cdot f(x + \Delta x, \Delta y, t) + \frac{\partial V}{\partial t} \\ &= \frac{\partial V}{\partial x} \cdot f(x, 0, t) + \frac{\partial V}{\partial t} + \\ &\quad + \frac{\partial V}{\partial x} \cdot (f(x + \Delta x, \Delta y, t) - f(x, 0, t)). \end{aligned}$$

Then, using (16) it results that

$$\dot{V}(x,t) \le -W_3(x) + \frac{\partial V}{\partial x} \cdot (f(x + \Delta x, \Delta y, t) - f(x, 0, t)).$$

Consider the sets $\Omega_{2_b} = \{x | W_2(x) \le b\}$ and $\Omega_{1_a} = \{x | W_1(x) \le a\}$. From the hypotesis made about a and b, it results that

$$\Omega_{2_b} \subset \Omega_{1_b} \subset \Omega_{2_a} \subset \Omega_{1_a} \subset D. \tag{17}$$

Since $W_3(x)$ is positive definite in D, it is positive in $\Omega_{1,2} \triangleq \Omega_{1_a} - \Omega_{2_b}$. Moreover, it exists a positive constant s where

$$s \triangleq \min_{x\in\Omega_{1,2}} W_3(x). \tag{18}$$

Let us define the following function

$$\begin{aligned}\alpha(x,\Delta x,\Delta y,t) \quad \triangleq \quad & -W_3(x) + \frac{\partial V}{\partial x}\cdot \\ & \cdot(f(x+\Delta x,\Delta y,t) - f(x,0,t)).\end{aligned} \tag{19}$$

The continuity in functions W_3 and f and the fact that V is continuously differentiable implies that α is uniformly continuous. From the definition of α and Eq.(18) it results that

$$\alpha(x,0,0,t) \leq -s \;\; \forall x \in \Omega_{1,2}, \forall t \geq 0. \tag{20}$$

Let α_M be the function defined by

$$\alpha_M(\Delta x,\Delta y) \triangleq \sup_{x\in\Omega_{1,2},t\geq 0}(\alpha(x,\Delta x,\Delta y,t)). \tag{21}$$

It can be easily seen that α_M is continuous and $\alpha_M(0,0) \leq -s$. Then, for any positive number $s_1 < s$ a positive constant r can be found so that the condition

$$\|(\Delta x,\Delta y)\| \leq r \tag{22a}$$

implies that

$$\alpha_M(\Delta x,\Delta y) \leq -s_1 \tag{22b}$$

and then it results that

$$\dot{V}(x,t) \leq \alpha(x,\Delta x,\Delta y,t) \leq \alpha_M(\Delta x,\Delta y) \leq -s_1 \tag{23}$$

in $\Omega_{1,2}$.

From here to the end, the proof follows Theorem 5.3 of (Khalil, 1996).

Let $\Omega_{a,t} = \{x|V(x,t) \leq a\}$ and $\Omega_{b,t} = \{x|V(x,t) \leq b\}$ be time variable sets. From (15) it results that

$$\Omega_{2_b} \subset \Omega_{b,t} \subset \Omega_{1_b} \subset \Omega_{2_a} \subset \Omega_{a,t} \subset \Omega_{1_a} \subset D \tag{24}$$

The border of $\Omega_{a,t}$ is inside $\Omega_{1,2}$, where $\dot{V}(x,t)$ is negative. It means that the trajectories of the QSC system (13) cannot abandon $\Omega_{a,t}$.

Then, any trajectory starting in Ω_{2_a} cannot abandon Ω_{1_a}.

The border of $\Omega_{b,t}$ is also inside $\Omega_{1,2}$. Then the trajectories cannot abandon this time variable set.

To complete the proof, we need to ensure that the trajectories initiated in $\Omega_{2_a} \subset \Omega_{a,t}$ reach $\Omega_{b,t}$ in a finite time.

Let $\phi(t)$ be a solution of (13) starting in $\Omega_{a,t}$ (i.e. $V(\phi(0),0) \leq a$) and let us suppose that

$$V(\phi(t),t) > b \;\; \forall t \tag{25}$$

then we have $\dot{V}(\phi(t),t) \leq -s_1$ and after

$$t_1 \triangleq \frac{a-b}{s_1} \tag{26}$$

it results that $V(\phi(t_1),t_1) \leq b$ which yields a contradiction. Then, the region $\Omega_{b,t}$ must be reached before the finite time t_1.

Since $\Omega_{b,t} \subset \Omega_{1_b}$ the trajectory also reaches the set Ω_{1_b} before that time. □

Equation (22a) gives the maximum perturbation allowed to ensure the achievement of the proposed goal (i.e. region of attraction Ω_{2_a} and ultimate bound in Ω_{1_b}). Since the maximum perturbation in each variable is given by the corresponding quantum, this equation should be used to choose the quantum at the different controller state variables and converters completing in that way the QSC design.

Observe that Ω_{2_a} is also the estimation of the region of attraction of the CCS using the Lyapunov function V. Then, a QSC implementation can be found so that it conserves the estimated region of attraction.

B. Design Algorithm for TV QSC

The design of a QSC contoller can be divided in two steps. The first one is the design of the continuous controller, which can be done following any technique.

The second step is the choice of the quantization at each variable. The use of a very small quantum yields solutions which are very closed to the trajectories of the CCS. This is due to the property of convergence which tells that –under locally Lipchitz conditions on the functions– the solutions of the QSC system go to the solutions of the CCS when the quantization go to zero (Kofman, 2003). In that way, and also according to Theorem 1, the ultimate bound can be reduced to arbitrary small values.

However, the use of a small quantum increases the number of events at the controller and the digital device can fail in its attempt to give the correct output values at the required time.

Therefore, there is always a trade–off between accuracy and practical considerations related to the computational costs. Then the idea is to exploit Theorem 1 in order to choose the quantization according to some essential features (region of attraction and ultimate bound). In that way, the quantization adopted will be just as small as necessary to ensure those properties and –provided that the CCS is not too fast– the digital device will be able to correctly implement the resulting QSC controller.

The translation of these ideas into a design algorithm for QSC can be written as follows:

1. Design a continuous controller and calculate the Lyapunov function $V(x,t)$ and the functions $W_i(x)$ according to (15)–(16) for the closed loop CCS.

2. Choose the QSC region of attraction Ω_{2_a} and the ultimate region Ω_{1_b} together with the constants a and b.

3. Obtain the perturbed closed loop function f according to (13).

4. Obtain function α according to (19) and α_M according to (21).

5. Calculate constant s according to (18) and choose the positive constant $s_1 < s$. If the goal is just to ensure ultimately boundedness s_1 should be very small. Otherwise, if the speed of convergence is also important, it can be chosen taking into account (26).

6. Compute the value of r according to (22).

7. Choose the quantization at the controller state variables and converters so that (22a) is satisfied.

It can be easily seen that this procedure leads to a QSC controller which ensures region of attraction Ω_{2_a} and ultimate region Ω_{1_b}.

IV. EXAMPLES AND RESULTS

The unstable time varying plant

$$\begin{cases} \dot{x}_p &= x_p \cdot (1 + \sin t + \cos t) + u_p \\ y_p &= (2 + \cos t) \cdot x_p \end{cases}$$

can be stabilized by the controller

$$\begin{cases} \dot{x}_c &= -x_c + u_c \\ y_c &= -x_c - u_c. \end{cases} \tag{27}$$

The resulting closed loop system can be written as

$$\begin{cases} \dot{x}_p &= -(1 - \sin t) \cdot x_p - x_c \\ \dot{x}_c &= (2 + \cos t) \cdot x_p - x_c. \end{cases}$$

Here, the Lyapunov candidate

$$V(x_p, x_c, t) = x_p^2 + \frac{1}{2}x_c^2 + \frac{1}{2}x_p^2 \cos t$$

verifies (15) with

$$W_1(x_p, x_c) = \frac{1}{2}x_p^2 + \frac{1}{2}x_c^2 \tag{28}$$

$$W_2(x_p, x_c) = \frac{3}{2}x_p^2 + \frac{1}{2}x_c^2.$$

The orbital derivative is

$$\dot{V} = (2 + \cos t) \cdot (\sin t - 1) \cdot x_p^2 - x_c^2 - \frac{1}{2}x_p^2 \sin t$$

which satisfies (16) with

$$W_3 = -\frac{1}{2}x_p^2 - x_c^2.$$

Then, the closed loop CCS is asymptotically stable and the algorithm resulting from Theorem 1 can be used to design the QSC controller.

The first step for the QSC design consists in choosing the region of attraction and the ultimate bound. Since inequalities (15)–(16) stand in $\Re^2$ (i.e. the CCS stability is global) it is not necessary to restrict the region of attraction except for chosing the saturation values. In this case, the choice of Ω_{2_a} does not affect the calculations.

The ultimate bound Ω_{1_b} will be chosen with $b = 0.5$. Then, taking into account (28) it results that $\Omega_{1_b} = \{x \in \Re^2 | \|x\| \leq 1\}$.

The perturbed equations (13) can be written as

$$\begin{cases} \dot{x}_p &= -(1 - \sin t) \cdot x_p - x_c - \Delta x_c + \Delta y_c - \Delta y_p \\ \dot{x}_c &= (2 + \cos t) \cdot x_p - x_c - \Delta x_c + \Delta y_p \end{cases}$$

and then, from (19) function $\alpha(x, \Delta x, \Delta y, t)$ results

$$\begin{aligned} \alpha &= -x_c^2 - \frac{1}{2}x_p^2 + x_p \cdot (2 + \cos t) \cdot \\ &\quad \cdot(-\Delta x_c + \Delta y_c - \Delta y_p) + x_c \cdot (-\Delta x_c + \Delta y_p). \end{aligned}$$

Although the maximum α_M in (21) cannot be easily obtained, it can be bounded using the fact that $\|x\| > 1/3$ in $\Omega_{1,2}$. Then, after some calculations it results that

$$\begin{aligned} \alpha_M &\leq -\frac{1}{18} + \big((|\Delta x_c| + |\Delta y_c| + |\Delta y_p|)^2 + \\ &\quad + \frac{1}{9}(|\Delta x_c| + |\Delta y_p|)^2\big)^{1/2}. \end{aligned}$$

Thus, taking the quantum equal to 0.018 in the controller and converters we can ensure that $\alpha_M \leq -0.0093$ in $\Omega_{1,2}$, which implies that the trajectories finish inside Ω_{1_b} in finite time, with a minimum *speed* $s_1 = 0.0093$.

Figures 2–5 show the simulation results for an initial condition $x_p = 5$, $x_c = 0$.

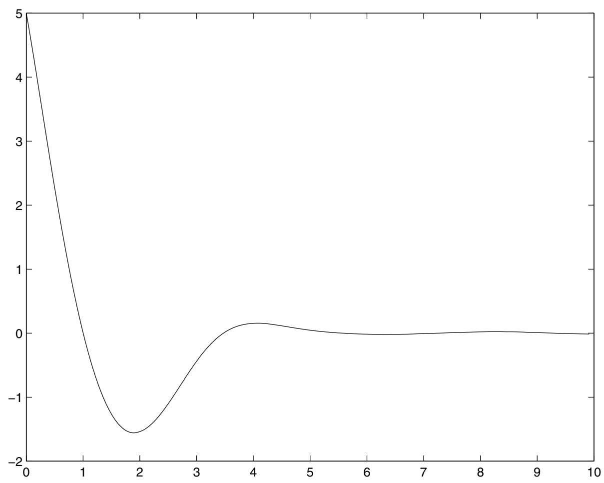

Figure 2. x_p vs. t

Figure 5 also shows that the design was very conservative. The ultimate bound observed in the simulation is less than 0.03 (in norm 2) which is more than 30 times smaller than the estimated.

There are two reasons which can explain this. The first one is that Lyapunov analysis often leads to conservative estimations of the ultimate bounds. The

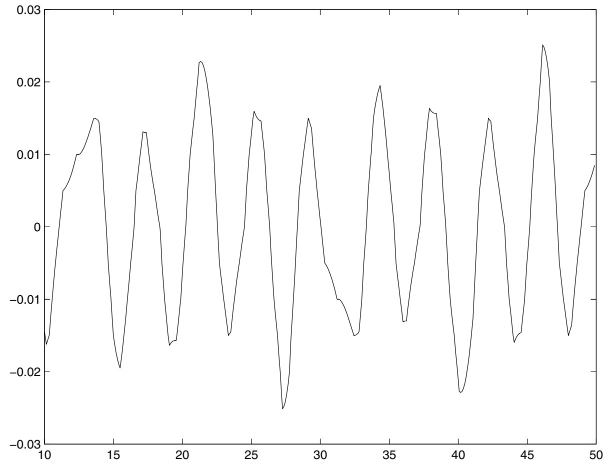

Figure 3. Final oscillations in x_p vs. t

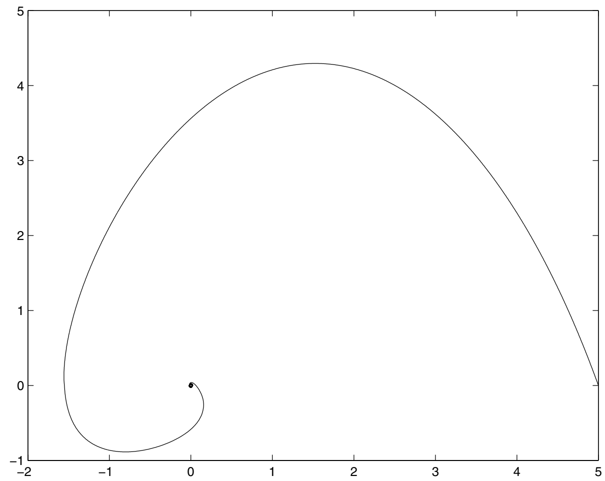

Figure 4. x_p vs. x_c

second reason is that in this case the D/A converter does not introduce any perturbation since it is exactly matched with the quantizer of x_c and the A/D converter.

V. CONCLUSIONS

This work studied the properties of QSC in time varying systems showing that the main features and advantages of the methodology are also satisfied in these cases.

Theorem 1 proved that the QSC control of time varying plants allows to ensure ultimately boundedness conserving the region of attraction.

A design algorithm derived from this theorem was also introduced and its use was illustrated with an example.

When it comes to future work, it should be considered that the main theoretical properties were already proven for general nonlinear time varying systems and for LTI systems. There is an intermediate case which should be taken into account which corresponds to Linear Parameter–Varying (LPV) plants. The example given in fact corresponds to that category

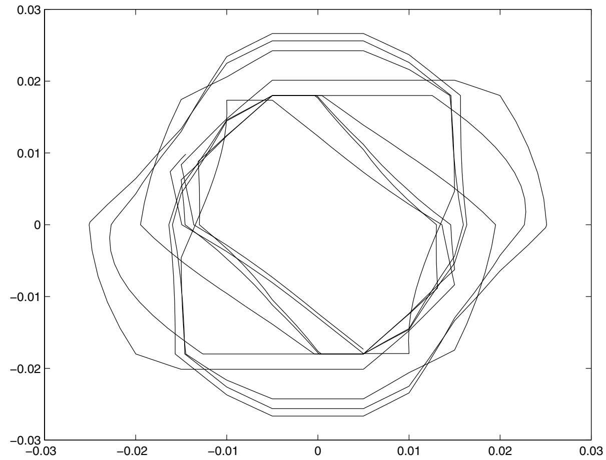

Figure 5. Final oscillations in x_p vs. x_c

and although a Lyapunov analysis could be done for that case, the result was very conservative. If the geometrical analysis for LTI systems of (Kofman, 2002) were extended for LPV plants, less conservative results might be obtained.

REFERENCES

Brockett, R. and D. Liberzon, "Quantized feedback stabilization of linear systems," *IEEE Trans. Automat. Contr.,* **45**, 1279–1289 (2000).

Delchamps, D., "Stabilizing a linear system with quantized state feedback," *IEEE Trans. Automat. Contr.,* **35**, 916–924 (1990).

Elia, N. and S. Mitter, "Stabilization of Linear Systems with Limited Information," *IEEE Trans. Automat. Contr.,* **46**(9), 1384–1400 (2001).

Hou, L., A. Michel, and H. Ye, "Some qualitative properties of sampled-data control systems," *IEEE Trans. Automat. Contr.,* **42**, 1721–1725 (1997).

Khalil, H., *Nonlinear Systems*, Prentice-Hall, New Jersey, 2nd edition (1996).

Kofman, E., "Quantized-State Control of Linear Systems," *Proceedings of AADECA 2002*, Buenos Aires, Argentina (2002).

Kofman, E., "Quantized-State Control. A Method for Discrete Event Control of Continuous Systems," *Latin American Applied Research,* **33**(4), 399–406 (2003).

Kofman, E. and S. Junco, "Quantized State Systems. A DEVS Approach for Continuous System Simulation," *Transactions of SCS,* **18**(3), 123–132 (2001).

Kofman, E., J. Lee, and B. Zeigler, "DEVS Representation of Differential Equation Systems. Review of Recent Advances," *Proceedings of ESS'01,* 591–595 (2001).

Miller, R., A. Michel, and J. Farrel, "Quantizer effects on steady-state error specifications of digital feedback control systems," *IEEE Trans. Automat. Contr.,* **34**, 651–654 (1989).

Sayiner, N., H. Sorensen, and T. Viswanathan, "A non-uniform sampling technique for A/D conversion," *Proceedings of ISCAS'93,* **2**, 1220–1223 (1993).

Zeigler, B. and J. Kim, "Extending the DEVS-Scheme knowledge-based simulation environment for real time event-based control," *IEEE Trans. on Robotics and Automation,* **3**(9), 351–356 (1993).

Zeigler, B., T. Kim, and H. Praehofer, *Theory of Modeling and Simulation. Second edition*, Academic Press, New York (2000).

Received: October 8, 2003.
Accepted for publication: October 27, 2004.
Recommended by Guest Editor O. Crisalle.