ROBOT CONTROL WITH INVERSE DYNAMICS AND NON-LINEAR GAINS

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Abstract—— A motion control strategy for robot manipulators, with inverse dynamics and non-linear proportional-derivative gains is presented. On account of a possible interaction of the robot with the environment, impedance is incorporated to modify the robot's motion references according to the interaction force. The gains, that are non-linear state functions, allow to improve robot performance and to prevent actuator saturation. It is proved that an asymptotically stable closed-loop system is obtained with the proposed controller. Simulation results on a 3-dof robot show a good performance of the controller with variable gains, as opposed to that of a constant gain PD controller.

Key words—— robotics, non-linear systems, motion control, impedance control.

I. INTRODUCTION

One of the basic problems in robot control is the so-called motion control, when a robot manipulator is required to follow a pre-established trajectory.

Current present-day manipulators use proportional-derivative controllers (PD) or proportional-integral-derivative controllers (PID) in closed-loop systems in order to reach the desired configurations. For an updated reference on PID controllers, see (Benett, 2001) and (Aström and Hägglund, 2001). It has been proven that PID controllers, despite their widespread use, do not show global asymptotic stability when controlling a robotic manipulator (Wen and Murphy, 1990). Various motion controllers with rigorous stability demonstrations can be found in the literature: (Sciavicco and Siciliano, 2000; Craig, 1989) among others.

The PD controller with gravity compensation produces a global asymptotically stable closed-loop system through a trivial selection of the proportional and derivative gains (Takegaki and Arimoto, 1981). The PD+ controller, introduced by Koditschek (Koditschek, 1984) is both simple and attractive. Its control structure is based on a linear PD feedback loop plus a specific compensation of robot dynamics. The first stability analysis of a PD+ controller was done by Paden and Panja (1988), who termed it PD+ control. Later, Whitcomb *et al*. (1993) present a rigorous stability analysis by introducing a Lyapunov function in an adaptive control context.

The global asymptotic stability analysis of a closed-loop system using the above-cited controllers has been carried out in the above mentioned papers by considering a selected set of constant gains of the controllers. This characteristic may constrain the application of these controllers when, in addition to asymptotic stability, a high performance of the control system is required as well. To have a good performance in manipulator control with actuator constrains implies to implement variable gains in the controllers. Variable-gain PD controllers for position and motion control of manipulators have been implemented in (Kelly and Carelli, 1996) and, recently, Santibañez *et al*. (2000) presented a variable-gain PD+ controller that uses fuzzy logic.

We present here an inverse dynamics controller with non-linear PD gains, which allows for motion and impedance control. It avoids saturation of control actions and improves the performance for small control errors.

The work is organised as follows. Section II describes the model and control scheme along with its stability analysis. The application of a control algorithm to a 3-dof manipulator is presented in Section III, and the conclusions in Section IV.

II. ROBOT MODEL AND CONTROLLER DESIGN

A Robot Model

With no perturbations present, the joint-coordinates dynamic model of a robot manipulator interacting with the environment is:

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + \theta(\dot{q}) + J^T(q) f_m \quad (1)$$

where τ is the $n \times 1$ vector of torques or forces applied on the joints, $M(q) \in \Re^{n\times n}$ is the manipulator's inertia matrix that is symmetric and positive definite; $C(q,\dot{q}) \in \Re^{n\times n}$ is the matrix of centrifugal and Coriolis forces; $g(q) \in \Re^{n}$ is the vector of gravitational torques or forces; $\theta(\dot{q})$ is the friction force vector; f_m is the vector of forces interacting with the environment and $q \in \Re^n$ is the joints position vector.

For an analysis of the interaction between manipulator and environment and, since this interaction is typically described in the operational space, it is convenient to refer the control schemes to such a space. Then, the dynamic model of the manipulator in Cartesian coordinates (Sciavicco and Siciliano, 2000) can be written as:

$$f_a = H^*(x)\ddot{x} + C^*(x,\dot{x})\dot{x} + g^*(x) + \Theta(\dot{x}) + f_m \quad (2)$$

where f_a is the actuator force defined at the end effector, H^* is the inertia matrix of the manipulator and $C^*\dot{x}$ the centrifugal and Coriolis forces, Θ is the friction force vector, g^* the gravity force vector; $x \in \Re^n$ is the position vector in Cartesian coordinates and f_m is the interaction force with the environment. This force, for an elastic environment, can be modelled by,

$$f_m = K_m(x - x_e) \quad (3)$$

where K_m is the environment's elasticity matrix and x_e is the environment's position vector. The dynamic model of Eqn. (2) is defined outside the singular configurations of the robot.

B. Control Strategy

A simplified scheme for the proposed control is shown in Fig. 1.

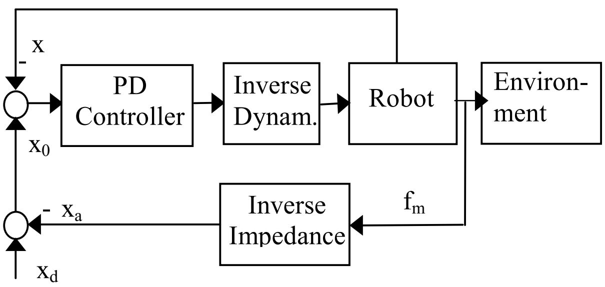

Figure 1 Control Scheme

The control law involves two feedback loops. One of them is external and generates a modified motion reference by adding a term obtained after filtering the interaction force by the inverse of the transfer function of the desired impedance (Carelli and Postigo, 1992). Besides, the modified reference is obtained for both the velocity and the acceleration. These modified references are applied to the internal loop.

The internal loop follows an inverse dynamics PD motion control law:

$$f_a = H^*(x)a + C^*(x,\dot{x})\dot{x} + g^*(x) + f_m \quad (4)$$

where $\tilde{x}$ is the error vector defined as: $\tilde{x} = x_o - x$, and $x_o = x_d - x_a$, with x_d the desired trajectory and the reference adjustment vector x_a obtained by applying the impedance inverse to the interaction force, by means of the following law:

$$x_a(t) = \left[Ap^2 + Bp + K\right]^{-1} f_m(t) \quad (5)$$

A, B, K are the matrices for the desired impedance, $p=d/dt$.

Note that if $f_m = 0$, i.e., no contact with the environment is verified, then $x_o = x_d$, thus the manipulator follows the nominal reference.

The vector a in (4) is the resultant of a proportional-derivative controller with non-linear gains, defined by,

$$a = \ddot{x}_o + K_v(\tilde{x},\dot{\tilde{x}})\dot{\tilde{x}} + K_p(\tilde{x})\tilde{x} \quad (6)$$

where K_p, K_v are positive definite matrices given by:

$$K_p(\tilde{x}) = diag\left(\left\lfloor K_{pi} \right\rfloor_{i=1\ldots n}\right) \quad (7)$$

with

$$K_{pi}(\tilde{x}) = \begin{cases} k_{pi}\left(\dfrac{tanh\left(\dfrac{k_{pi}}{k_{i_{max}}}\tilde{x}_i\right)}{\dfrac{k_{pi}}{k_{i_{max}}}\tilde{x}_i}\right) & \tilde{x}_i \neq 0 \\ k_{pi} & \tilde{x}_i = 0 \end{cases} \quad (8)$$

and matrix K_v is given by

$$K_v(\tilde{x},\dot{\tilde{x}}) = diag\left(\left[K_{vi}\right]_{i=1\ldots n}\right), \quad (9)$$

with

$$K_{vi}\left(\tilde{x},\dot{\tilde{x}}\right) = \begin{cases} 2\sqrt{k_{pi}}\sqrt{P}\,\dfrac{tanh\left(\dot{\tilde{x}}_i\right)}{\dot{\tilde{x}}_i}, & \left(\tilde{x}_i,\dot{\tilde{x}}_i\right) \neq 0 \\ 2\sqrt{k_{pi}} & \left(\tilde{x}_i,\dot{\tilde{x}}_i\right) = 0 \end{cases} \quad (10)$$

where $$P = \frac{tanh\left(\dfrac{k_{pi}}{k_{i_{max}}}\tilde{x}_i\right)}{\dfrac{k_{pi}}{k_{i_{max}}}\tilde{x}_i}$$

where k_{pi} are constants to be determined in such a way that when $\tilde{x}_i \to 0$ the controller will ensure that the robot follows the desired trajectory without saturating the actuators. The constants $k_{i_{max}}$ should be chosen in such a way that when $\tilde{x}_i$ increases, the value of a remains bounded. Thus, the manipulator can follow the trajectory without going beyond the maximum values allowed for the actuators, provided the robot is moving within a region far enough from singularities. Then, to compute $k_{i_{max}}$ it is necessary to know the maximum acceleration a_{max} attainable by the robot, or at least an estimation of this value from simulations. Then, knowing a_{max} and applying limits to Eqn. (6) when $\tilde{x}_i \to \infty$ y $\dot{\tilde{x}}_i \to \infty$ it follows that

$$k_{i_{max}} + 2.\sqrt{k_{i_{max}}} - \left(a_{max} - \ddot{x}_0\right) = 0 \qquad (11)$$

where $\ddot{x}_o$ depends on the followed trajectory. By solving the quadratic equation, the positive value of the root is chosen. After the above-mentioned definition of constants k_{pi}, both K_p, K_v are positive definite matrices.

Remark 1: in Eqns. (8) and (10), the *tanh* non linear function has been used, due to its good properties such as to be bounded between 1 and -1 with unity slope close to 0, continuous and odd. Note that any other function with similar properties could be also used in defining the variable controller gains. □

A demonstration below will show that the proposed controller is asymptotically stable. The following lemma will be used

Lemma 1: If $f:\Re \to \Re$ is an increasing, odd, continuous function then

$$\int_0^x f(u)du > 0 \qquad \forall x \in \Re . \qquad (12)$$

Proposition 1: Let the dynamic model of a robot be stated by Eqns. (4) and (6), where the elements of the gain matrices of the PD controller are defined by Eqns. (8) and (10). Then, the closed-loop system is asymptotically stable, and it is verified that

$$\lim_{t\to\infty} x(t) = x_d(t) \qquad (13)$$

Proof: The closed-loop system obtained by combining the robot model (4) with the control law (6) can be expressed as

$$\ddot{\tilde{x}} + K_v(\tilde{x},\dot{\tilde{x}})\dot{\tilde{x}} + K_p(\tilde{x})\tilde{x} = 0 \qquad (14)$$

A Lyapunov candidate function is proposed as

$$V = \frac{1}{2}\dot{\tilde{x}}^T\dot{\tilde{x}} + \int_0^{\tilde{x}} \mu^T K_p(\mu)d\mu \qquad (15)$$

with a positive first term representing the norm of the velocity error vector. The second term can be expressed

$$\int_0^{\tilde{x}} \mu^T K_p(\mu)d\mu = \sum_{i=1}^{n} \int_0^{\tilde{x}_i} \mu_i K_{pi}(\mu_i)d\mu_i \qquad (16)$$

Each of the summation terms is

$$\int_0^{\tilde{x}_i} \mu_i K_{pi}(\mu_i)d\mu_i = \int_0^{\tilde{x}_i} \mu_i k_{pi} \left(\frac{tanh\left(\frac{k_{pi}}{k_{i_{max}}}\mu_i\right)}{\frac{k_{pi}}{k_{i_{max}}}\mu_i} \right) d\mu_i \qquad (17)$$

$$\int_0^{\tilde{x}_i} \mu_i K_{pi}(\mu_i)d\mu_i = \int_0^{\tilde{x}_i} \mu_i k_{imax} tanh\left(\frac{k_{pi}}{k_{i_{max}}}\mu_i\right) d\mu_i \qquad (18)$$

and, bearing in mind that *tanh* is a continuous, odd and increasing function, by Lemma 1 it is verified that the integral (17) is positive. Then, function V is positive definite. As regards the time derivative $\dot{V}$, it follows:

$$\dot{V} = \dot{\tilde{x}}^T\ddot{\tilde{x}} + \tilde{x}^T K_p(\tilde{x})\dot{\tilde{x}} \qquad (19)$$

Then, by substituting the closed-loop equation

$$\dot{V} = -\dot{\tilde{x}}^T K_v(\tilde{x},\dot{\tilde{x}})\dot{\tilde{x}} \qquad (20)$$

and, since by definition K_v is positive definite, we conclude that $-\dot{V}$ is positive semi-definite. It should be resorted, then, to apply the Krasovski-Lasalle theorem (Vidyasagar, 1993). It can be verified that, within the set

$$\Omega = \left\{ \begin{bmatrix} \tilde{x} \\ \dot{\tilde{x}} \end{bmatrix} \Big/ \dot{V}(\tilde{x},\dot{\tilde{x}}) = 0 \right\} = \left\{ \dot{\tilde{x}} = 0 \right\} \qquad (21)$$

the origin is the unique invariant set. This can be verified by noting in the dynamic Eqn. (14) that, when $\dot{\tilde{x}} = 0$, the unique solution for $\ddot{\tilde{x}} = 0$ is $\tilde{x} = 0$. Therefore, the origin of the space state is asymptotically stable, thus verifying Eqn. (13).

Remark 2: For the robot dynamics model as the one described in Eqn. (2), the analysis is valid outside the singular configurations of the robot, due to well known restrictions of inverse kinematics.

III . SIMULATION RESULTS

A model of PUMA 560 robot with 3-dof (waist, shoulder and elbow) (Troch and Desoyer, 1990) was used. Figure 2 shows a scheme of this robot.

The planned task is to polish a plane surface parallel to the x-y plane of the reference base system, following a trajectory defined by Santibañez *et al.* (2000):

$$
\begin{cases}
x_d = 0.3 + 0.05(1 - e^{-2t^4})\sin(t) \\
y_d = 0.2 - 0.05(1 - e^{-2t^4})\sin(t) \\
z_d = z_0
\end{cases}
\quad (22)
$$

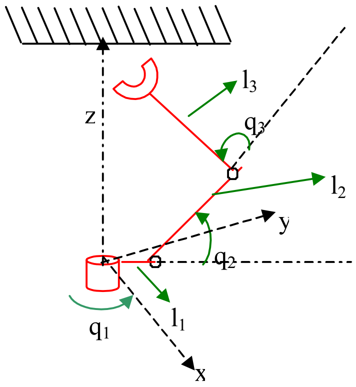

Figure 2 Robot scheme

with initial position at $x_0 = 0$; $y_0 = 0.22$m z_0=0.56m.

The proposed inverse dynamics, non-linear gain PD controller was implemented, and the results obtained were contrasted with those of an inverse dynamics, constant gains PD controller. Gains for the second controller are listed in Table 1. For the variable gain controller, it was chosen $k_{pi} = 180$ and $a_{max} = [1.5 \ \ 10 \ 10]$. The value of $k_{i_{max}}$ was computed with Eqn. (11).

Table1. Data of the Constant Gain Controller

P-D gains	Impedance
$K_p = diag([20;20;20])$	$A = 1$ $B = 10$
$K_v = 2\sqrt{K_p}$	$K = 25$

The position errors obtained with both controllers are presented in Figs. 3 and 4. It can be noted that the constant-gain PD controller can not follow the trajectory with zero error. It oscillated about a constant error zone, because an increase in the gains saturates the actuators, as in Fig. 5. The variable-gain PD controller accomplished the trajectory without saturating the actuators, as shown in Fig. 6. Also, a good performance for the system is obtained. The reasons are that the gains decrease when the error is high, and they increase when the error tends to zero (see Fig. 7). Actuator saturation is thus avoided. The presence of non-modelled non-linear dynamics causes that the constant-gain proportional derivative controller does not reach the desired trajectory, whereas the proposed controller does attain the objective with a high performance, because it allows to increase the gains when the error is small without saturating the actuators. The non-modelled dynamics in this simulation corresponds to Coulomb's friction effects.

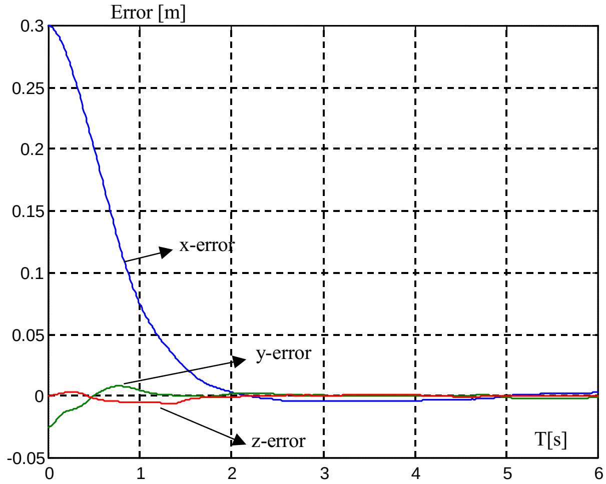

Figure 3: Position errors with variable gains

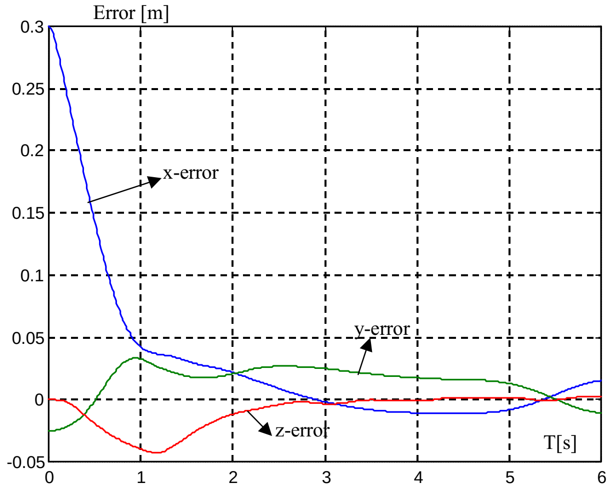

Figure 4: Position errors with constant gains

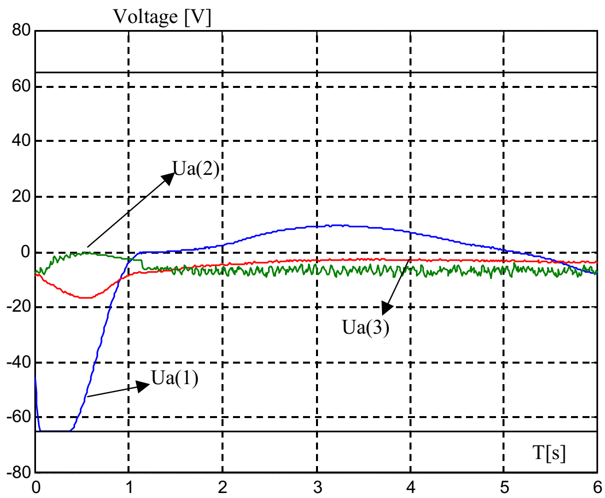

Figure 5: Voltages obtained with the constant-gain controller

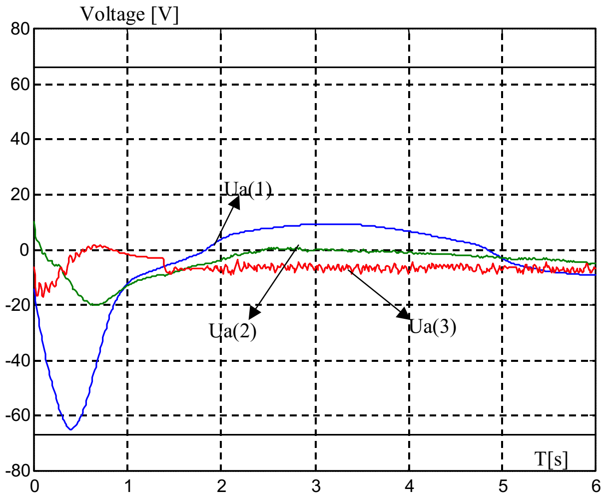

Figure 6: Voltages obtained with the variable-gain controller

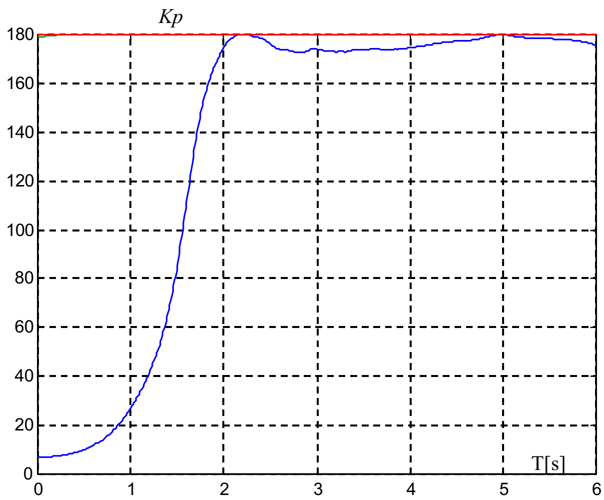

Figure 7: Variable Gains K_p of the controller

IV. CONCLUSIONS

This work has presented a control strategy for robot manipulators that allows to improve the performance and to avoid saturation of the control actions. The control is composed of an inverse-dynamics controller with PD gains that are non-linear state functions. It has been proven that the proposed controller is asymptotically stable. The simulation results show a good behaviour of the controller, which has reached the proposed objectives. It should be observed that, even though the work is referred to robot manipulators, the proposed controller can be applied to any second order non-linear system which allows for feedback linearization.

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