REGULAR ARTICLES

TRANSIENT NATURAL CONVECTION IN A HORIZONTAL FLUID LAYER, WITH A BLACKBODY BOTTOM; HEATED FROM ABOVE BY RADIATION

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Abstract **- Transient natural convection in a horizontal fluid layer, with a blackbody bottom, heated from above by radiation was studied experimentally and numerically. The heat flux which reaches to the bottom is absorbed by the black surface and reheats the fluid layer from the bottom, creating an adverse temperature gradient at the bottom of the fluid layer similar to the one observed in the classical Rayleigh-Bénard problem.**
A mathematical model was developed and governing differential equations were numerically solved. The predictions of the model were found to be in an acceptable agreement with the experimental temperature distributions obtained in a laboratory scale pool by holographic interferometer techniques.
Experimentally and numerically calculated heat convection coefficients were correlated in terms of Nusselt and Rayleigh numbers and compared with a literature correlation. It is concluded that this problem is very similar to the classical Rayleigh-Bénard problem of a horizontal fluid layer heated from below.

Keywords **- Transient natural convection, Rayleigh-Bénard problem, thermal instability, holographic interferometer.**

I. INTRODUCTION

Natural convection heat transfer in horizontal fluid layers has been widely investigated by number of researchers. In most of the cases the layer is heated from the bottom and cooled from the top. The first experimental studies were carried out in the beginning of the twentieth century (Bénard, 1901) and the theoretical ones were realized few years later (Rayleigh, 1916). Extended summaries of various studies, under different boundary conditions, by various researchers are presented by Gershuni and Zhukhovitskil (1972) as well as Chandrasekkar (1961).

In general, a horizontal fluid layer heated from the bottom is inherently unstable. Heat is transferred through conduction up to some certain temperature difference and in this regime the fluid is stable. After a critical value of temperature difference the fluid particles show a tendency to move vertically. At this point onset of convection is established. In convection regime some hexagonal cells, called Bénard cells, are formed (Bénard, 1901). After this point the movements of particles depend on two forces competing each other; buoyancy forces and viscous forces. Practical applications of this problem are seen in cooling of electronic equipments and thermal storage tanks.

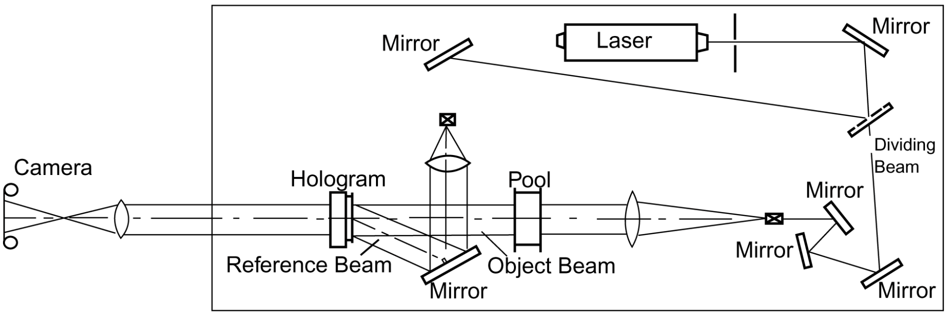
Enhancement in Rayleigh-Bénard convection was studied by Domaradzki (1989), showing that with proper forcing, it is possible to control the size of convection cells. Hernández (1995), studied numerically the influence of heat transfer rate over the flow transition. Cerisier *et al.* (1998) analyzed the onset of convection in a horizontal fluid layer between two plates, with different thicknesses and thermal conductivities, and found a good agreement between his model and experiments. Thermal instability of fluids with high Prandtl numbers was experimentally studied by Wakitani (1994). Prakash and Koster (1996) studied the Rayleigh-Bénard problem in a system of two immiscible fluids. They observed that the convection in the two-layer system is characterized by two distinct coupling modes between the layers. They are thermal coupling and mechanical coupling.

On the other hand, in case of heating from above, if the bottom of the fluid layer is a blackbody, the heat flux which reaches to the bottom is absorbed by the blackbody bottom and reheats the fluid layer from the bottom, creating an adverse temperature gradient as the one encountered in the classical Rayleigh-Bénard problem of thermal instability. This is the phenomenon seen in solar ponds. Kozanoglu (1993) observed that, in case of heating from above, the instability phenomenon occurring at the bottom of the fluid layer with a blackbody bottom is very similar to the one in classical Rayleigh-Bénard problem and the experimental critical Rayleigh number is found to be very close to the theoretical critical Rayleigh number given in the literature.

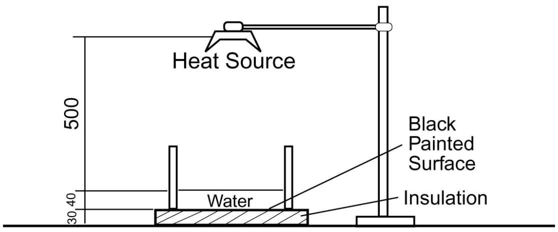
II. EXPERIMENTS

Transient natural convection in a horizontal fluid layer with a blackbody bottom, heated from above was first studied experimentally. The holographic interferometry set-up shown in Fig. 1 was used to find time dependent temperature profiles through the fluid layer.

The experiments were carried out in a laboratory scale pool, with dimensions of 200mm x 70mm x 180mm. The bottom of the pool was made of 10 mm. thick aluminum, which is painted black, while

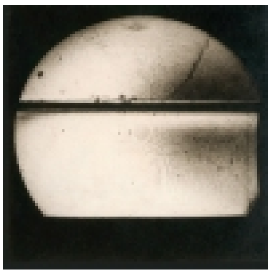


(a) Holografic Set - Up



(b) Laboratory Scale Pool

Figure 1. Experimental Equipment.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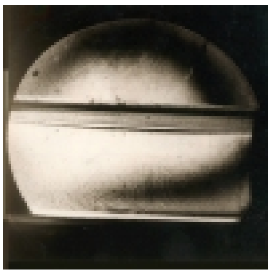
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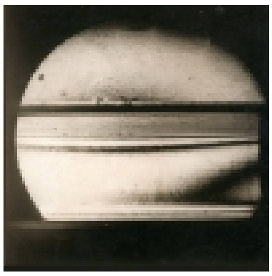
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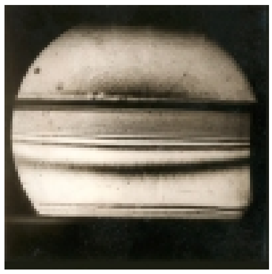
(d)


(e)


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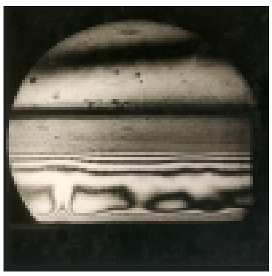
(g)


(h)

Figure 2. Photographs taken by holographic interferometer techniques at various instants during the heating process. a) heating starts b) t=15s., Ra_L=14 c) t=45s., Ra_L=503 d) t=66s., Ra_L=890 e) t=72s., Ra_L=Ra_{crit}, transition starts f) t=87s., Ra_L=1215 g) t=136s., Ra_L=11225 h) t=150s., Ra_L=17281.

the sidewalls of the pool were constructed of glass. The bottom of the pool was insulated by 30mm thick polyurethane layer to avoid heat losses. Purified water was used in the majority of experiments. Before the experiments, the fluid was boiled approximately one hour and then cooled to eliminate small air bubbles in it.

The fluid was heated by an ISING-2700 type lamp of a maximum power of 1200W. The lamp was placed 50 cm. above the fluid layer as shown in Fig. 1. The radiative heat energy reaching to the fluid surface from the lamp was measured by a Kipp-Zonen-CCI type pyronometer.

A series of experiments were carried out employing the holographic equipment by real-time technique. For various radiative heat fluxes, the temperature distributions through the fluid layer were measured and critical Rayleigh numbers were evaluated. Figure 2 shows a series of photographs taken by holographic interferometer techniques at different instants of an experiment. Transient experimental temperature distributions were obtained by evaluating these photographs over a photometer by considering that each interferometric line corresponds to a certain temperature difference. Experimental temperature distributions for the photographs shown in the Fig. 2 are presented in Fig.3. As observed in Fig. 2 (e), the interferometric lines at the bottom of the fluid layer show some waving motions at 72 seconds. These motions indicate beginning of instability and onset of convection.

The curves before 72 seconds correspond to conduction regime, while after this moment the convection regime is encountered. In all instants, the adverse temperature gradient created by the blackbody bottom is observed.

In case of natural convection from a horizontal plate, the characteristic length is a widely disputed concept and has been taken in distinct forms by different authors. In the classical Rayleigh-Bénard problem, the characteristic dimension, L, is taken as the thickness of the fluid layer where an adverse temperature gradient exists, while ΔT is the temperature difference through L (Rayleigh, 1916; Gershuni and Zhukhovitskil, 1972; and Chandrasekkar, 1961). This length is the distance through which particles of the fluid start to move upward as a result of density difference, when buoyancy forces overcome viscous forces. In the thermal instability part of this work the critical Rayleigh numbers were evaluated based on this characteristic length and found to be between 1026 and 1051, while the theoretical critical Rayleigh number for a horizontal layer bounded by one rigid and one free surface is given as 1100.65 (Chandrasekkar, 1961). The Rayleigh numbers presented in Figure 2, RaL, are calculated employing this characteristic length.

On the other hand the authors focused on the heat convection coefficient, rather than the thermal instability problem, defined different characteristic lengths. Fujii and Imura(1972) took the characteristic length as the width of the plate(smaller dimension), while McAdams (1954), Lloyd and Moran(1974), andGoldstein *et al.*(1973) suggested a characteristic length of $L=A'/P$, where A' and P are the area and the perimeter of the surface, respectively.

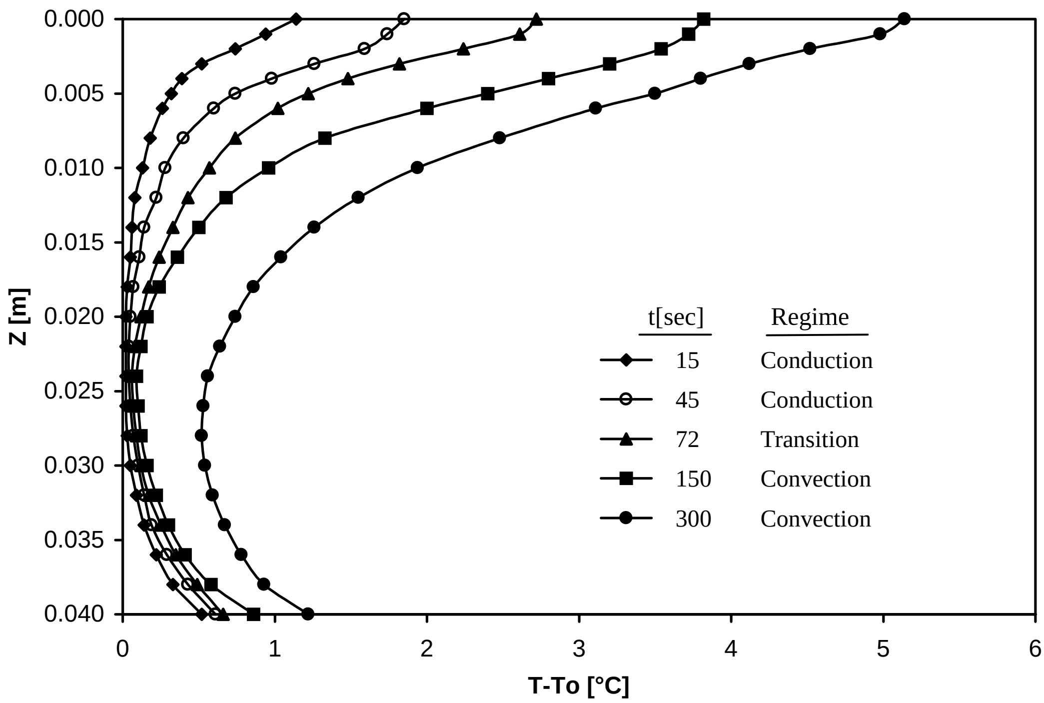

Figure 3. Experimental temperature distributions at various instants as a function of vertical distance.

In lack of a physically meaningful and commonly agreed characteristic length, in this study the width of the plate(smaller dimension) was decided to be taken as the characteristic length for the expressions of the heat convection coefficient, to be able to have a base to compare with the correlation proposed by Fujii and Imura(1972).

III. MATHEMATICAL MODEL

In the following part of the work, a mathematical model was developed considering energy balance of the infinitesimal element of dz shown in Fig. 4. A fraction of the radiactive energy, A, is reflected from the surface. Another fraction, β, is hold

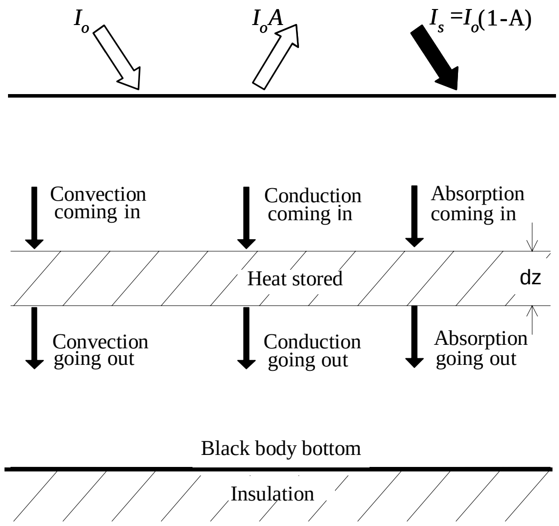

Figure.4 Scheme of the mathemathical model.

on the surface and conducted through the fluid layer. The rest of the radiative energy is partly absorbed through the layer according to Bourge's law. Under these considerations the following partial differential equation is obtained,

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} + \overline{w}\frac{\partial T}{\partial z} + \frac{I_s b}{\rho c_p}(1-\beta)e^{-bz} \tag{1}$$

where,

$$I_S = (1-A)I_O \tag{2}$$

The definition of the problem is completed by the following initial condition,

$$T(z, 0) = T_o \tag{3}$$

and boundary conditions,

$$I_S \beta A = -kA \left.\frac{\partial T}{\partial z}\right|_{z=0} \tag{4}$$

$$T(h,t) = T_o \tag{5}$$

The part of the radiation that reaches to the bottom of the layer and is absorbed by the blackbody bottom is expressed as,

$$I_{bottom} = I_S(1-\beta)(e^{-bh}) \tag{6}$$

In Eqn. 1, I_S is replaced by I_{bottom} and the equation is solved together with the same initial and boundary conditions to evaluate the temperature distribution created by the heating effect from the bottom. Then, the overall temperature distribution is obtained by superposing the temperature distributions from the top and the bottom.

IV. RESULTS

In the conduction regime, the only unknown is β, the fraction of the radiation hold on the surface, since $\overline{w}$, average convection velocity, is null. A non-linear regression code was developed to evaluate the values of β. The code combines some IMSL libraries with a differential system solver, DSS/2 (Pirkle and Schiesser, 1987). The system of partial differential equations was numerically solved using the method of Runge-Kutta, combined with the numerical method of lines, for a given value of parameter. Then, the regression model, by comparing the solutions of the model equations with experimental data, optimizes the parameter. β is a measure of the longwave fraction in the spectrum of the radiation that is incident over the fluid surface. Therefore, it does not vary in the convection regime. Using the value of β obtained in analysis of conduction regime, $\overline{w}$ appears as the only unknown in Eqn 1. Namely, there is only one fit parameter in each regime.

The simulations in the conduction regime resulted an average value of β=0.71 with a standard deviation of 3%. A very close value of β=0.70 was suggested by Dake and Harleman (1969). In case of convection regime the simulations provided an average value of the convection velocity in the range of 10^{-4} a 10^{-5} m/s. Figure 5 shows a comparison of the model predictions with experimental results at 15 seconds in conduction regime. Similar comparisons are presented in Fig. 6 and Fig. 7 for transition and convection regimes, respectively. As observed in these figures, a reasonable degree of agreement has been achieved between the experimental results and model predictions in all regimes. For a wide variety of operating conditions, a similar agreement has been obtained.

A parametric equation in terms of Rayleigh and Nusselt numbers was developed using experimentally calculated heat convection coefficients between the bottom surface of the pool and the fluid,

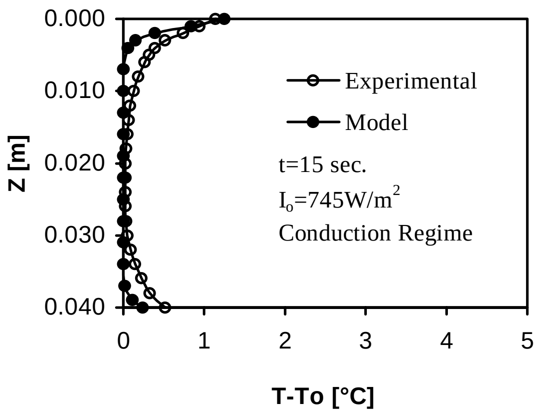

Figure 5. Comparison of the model predictions with experimental temperature distribution at 15 seconds, in conduction regime.

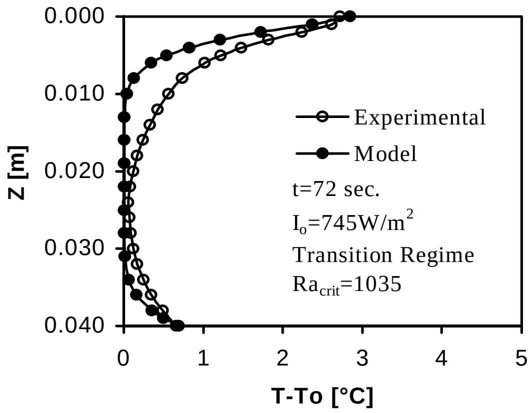

Figure 6. Comparison of the model predictions with experimental temperature distribution at 72 seconds, in transition regime.

$$Nu=1.25\ Ra^{0.18} \tag{7}$$

In this work, values of Rayleigh number never exceeded $1.1x10^7$. Then, the flow was always laminar. On the other hand, the model predictions for the same heat convection coefficient were correlated in a similar manner,

$$Nu=0.12\ Ra^{0.31} \tag{8}$$

In Fig. 8, Eqns (7) and (8) are compared with a literature correlation, Fujii and Imura (1972),

$$Nu=0.13\ Ra^{0.33} \tag{9}$$

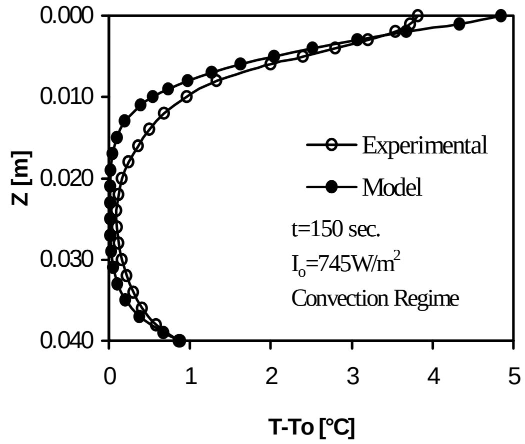

Figure 7. Comparison of the model predictions with experimental temperature distribution at 150 seconds, in convection regime.

As seen in this figure all the equations provide values of the same order of magnitude, while the model predicts lower values than Fujii and Imura (1972) and

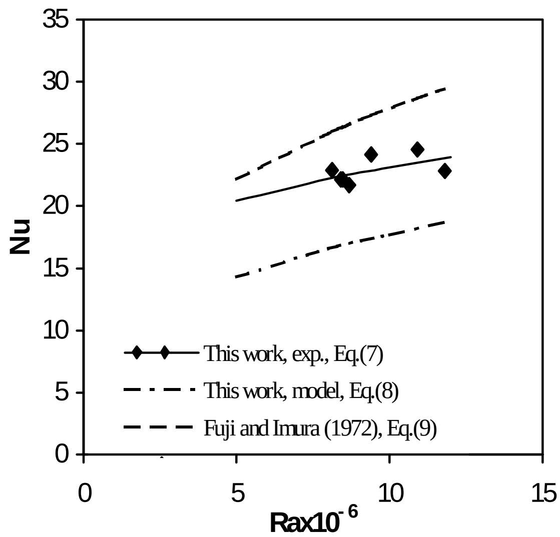

Figure. 8 Comparison of Nusselt numbers.

the experiments. This tendency can be attributed to the characteristics of the model that assumes a constant value of the fluid velocity. A further study, including the variation of velocity, is on the way to improve the model predictions.

V. CONCLUSIONS

The proposed mathematical model predicts well temperature distributions in a horizontal fluid layer, with a blackbody bottom, heated from above by radiation, in regime of conduction as well as convection. The studied problem was found to be very similar to the classical Rayleigh-Bénard problem of a direct heating from below. The heat convection coefficients calculated experimentally and numerically were found to be in a reasonable agreement with each other as well as with the values provided in the technical literature.

NOMENCLATURE

A	fraction of radiation reflected from the surface [-]
A´	area of the surface [m^2]
b	absorption coefficient [1/ m]
c_p	specific heat [kJ/kg°C]
d	width of the plate [m]
g	gravitational acceleration [m/s^2]
h	depth of fluid layer [m]
h´	convection coefficient [W/ m^2°C]
I_{bottom}	intensity of radiation reaching to the bottom of the fluid layer[W/m^2]
I_o	intensity of radiation [W/m^2]
I_s	intensity of radiation arriving to the surface of the fluid layer [W/m^2]
k	thermal conductivity [W/m°C]
L	characteristic dimension [m]
P	perimeter of the plate [m]
T	temperature [°C]
T_o	initial temperature [°C]
t	time [s]
$\bar{w}$	average fluid velocity [m/s]
z	vertical distance [m]

GREEK LETTERS

α	thermal diffusivity [m^2/s]
β	fraction of radiation hold on the surface [-]
β'	thermal expansion coefficient [1/°C]
ν	kinematic viscosity [m^2/s]
ρ	density of water [kg/m^3]

NONDIMENSIONAL GROUPS

Nu	Nusselt number [h´L/ k]
Ra	Rayleigh number [$g\beta'\Delta T\, d^3/\nu\alpha$]
Ra_L	Rayleigh number in thermal instability [$g\beta'\Delta T\, L^3/\nu\alpha$]

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