STRESS PROFILE MEASUREMENT IN AXIALLY SYMMETRIC GLASS SAMPLE

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The method based on the Senarmont compensation was proposed enabling the determination of the radial distributions of the axial, tangential and radial stress components in the cylindrical glass sample. The phase difference dependence on the radial coordinate was measured for light propagation through the sample in the axial and perpendicular (with respect to the axis of symmetry) direction. The numerical analysis based on the Taylor expansion of the radial distribution of particular stress components was used in combination with the linear regression analysis for evaluation of the stress components radial profiles and their confidence limits. Statistical treatment of experimental data obtained proved that when the cylindrical symmetry of the stress distribution is retained the Taylor expansion up to quadratic terms is sufficient for the description of stress distribution. The 95% confidence intervals of the obtained stress distributions are sufficiently narrow, corresponding to the relative error of the particular stress components values on the level of 5-10%.

INTRODUCTION

Stress identification in glass products contributes to proper setup of forming process and also increases product safety. Distinguishing tensile and compressive stress in three dimensions is still a challenge especially in items with complicated geometry.

Most of glass stresses measuring methods are using the fact that glass under stress separates the polarized light into two beams. Their amplitude difference can be detected as proportional to stress according to Brewster law [1-3]. Senarmont method is one of the most reliable methods enabling detection of the phase difference between ordinary and extraordinary light beam caused by the stress in glass.

Glass cylindrical probe is investigated in this paper. Measured stresses are described by polynomial functions excluding odd terms in order to express symmetry of the radial stress profiles. Photo-elastic approach [1-3] is completed with statistical evaluation in order to estimate parameters needed for individual component stress radial profile calculation.

Model

Senarmont compensation method offers the phase difference Δ defined by the integral of the difference between the principal stresses σ_1 and σ_2 (defined in the plane perpendicular to the light beam direction) along the pathway l of the light beam through the sample [2, 3]:

$$\Delta = B.\int_0^d (\sigma_1(l) - \sigma_2(l)).\mathrm{d}l \tag{1}$$

where d is the thickness of the sample, and B is the photo-elasticity constant. Supposing the value of B is known, the mean value of the main stresses difference $\sigma^{exp} = \Delta/B.d$ can be considered as a result of the measurement. Let us suppose the measurement is realized in the axial direction with the light beam passing the cylindrical sample of radius R at the distance ρ ($0 \leq \rho \leq R$) from the rotational axis of symmetry possessing the values denoted as $\sigma_a^{exp}(\rho)$. Analogously for the direction perpendicular to the rotational axis we obtain the values of $\sigma_p^{exp}(r)$ with the r value $0 \leq r \leq R$) defined according to the Figure 1. Denoting the radial dependences of the tangential, radial, and axial stress components as $\sigma_t(\rho)$, $\sigma_r(\rho)$, and $\sigma_a(\rho)$, and using the notation introduced in the Figure 1 we obtain:

$$\sigma_a^{exp}(\rho) = \sigma_t(\rho) - \sigma_r(\rho) \tag{2}$$

$$\sigma_p^{exp}(r) = \frac{1}{\sqrt{R^2 - r^2}} \int_0^{\sqrt{R^2-r^2}} f(y)\,\mathrm{d}y \tag{3}$$

where $f(y) = \sigma_a(\rho) - \sigma_r(\rho)\cos^2\alpha - \sigma_t(\rho)\sin^2\alpha =$

$$= \sigma_a\left(\sqrt{r^2+y^2}\right) - \frac{r^2}{r^2+y^2}\sigma\left(\sqrt{r^2+y^2}\right) - \frac{y^2}{r^2+y^2}\sigma_t\left(\sqrt{r^2+y^2}\right) \tag{4}$$

To evaluate the expressions from the Equations (2) and (3) we need some analytical expression of the individual stress profiles. A suitable representation is given by the Taylor polynomial series. Due to the fact that the stress profiles are symmetrical with respect to the $r = 0$ point, only the even power terms are retained in the Taylor series. Thus neglecting the terms of sixth and higher powers we obtain:

$$\sigma_t(\rho) = t_0 + t_2\rho^2 + t_4\rho^4 \tag{5}$$

$$\sigma_r(\rho) = r_0 + r_2\rho^2 + r_4\rho^4 \tag{6}$$

$$\sigma_a(\rho) = a_0 + a_2\rho^2 + a_4\rho^4 \tag{7}$$

Due to the following boundary conditions the unknown coefficients t_i, r_i, and a_i are not independent: In the center the radial and axial stress components are equivalent:

$$\sigma_r(0) = \sigma_t(0) \tag{8}$$

thus $r_0 = t_0$.

At the sample surface the radial stress component vanishes:

$$\sigma_r(R) = 0 \tag{9}$$

thus

$$r_2 = -\frac{t_0}{R^2} - r_4R^2 \tag{10}$$

and according to the Equation (2) $\sigma_a^{exp}(R) = \sigma_t(R)$ giving:

$$t_2 = \frac{1}{R^2}\left(\sigma_a^{exp}(R) - t_0 - t_4R^4\right) \tag{11}$$

Moreover in the case of perpendicular measurement at the sample surface the light beam is perpendicular to axial stress component and parallel to the radial one. Taking into account the Equation (9) we obtain:

$$\sigma_p^{exp}(R) = \sigma_a(R) \tag{12}$$

and consequently

$$a_0 = \sigma_p^{exp}(R) - a_2R^2 - a_4R^4 \tag{13}$$

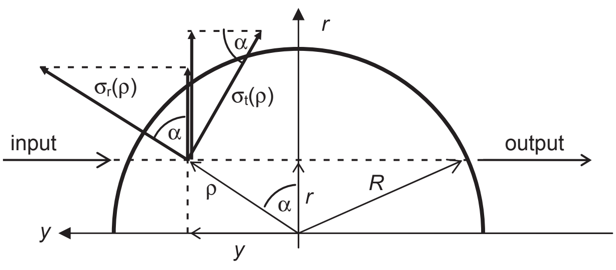

Figure 1. Stress decomposition during perpendicular measurement in immersion liquid.

Next the mechanical equilibrium constrains the ai coefficients:

$$\int_0^R \sigma_a(r) r\,\mathrm{d}r = \frac{1}{2}a_0R^2 + \frac{1}{4}a_2R^4 + \frac{1}{6}a_4R^6 = 0 \tag{14}$$

Combining the Eqs. (13) and (14) one obtains:

$$a_2 = \frac{2\sigma_p^{exp}(R)}{R^2} - \frac{4}{3}a_4R^2 \tag{15}$$

and

$$a_0 = -\sigma_p^{exp}(R) + \frac{1}{3}a\ R^4 \tag{16}$$

Taylor series for the individual stress components now contain only four unknown parameters, i.e. t_0, t_4, r_4, and a_4:

$$\sigma_r(\rho) = t_0\left(1 - \frac{\rho^2}{R^2}\right) + r_4\left(\rho^4 - R^2\rho^2\right) \tag{17}$$

$$\sigma_t(\rho) = t_0\left(1 - \frac{\rho^2}{R^2}\right) + t_4\left(\rho^4 - R^2\rho^2\right) + \sigma_a^{exp}(R)\frac{\rho^2}{R^2} \tag{18}$$

$$\sigma_a(\rho) = \sigma_p^{exp}(R) + \left(\frac{2\sigma_{aR}^{exp2}}{R^2} - \frac{4a_4R^2}{3}\right)\left(\rho^2 - R^2\right) + a_4\left(\rho^4 - R^4\right) \tag{19}$$

The remaining unknown parameters can be evaluated by the least squares fitting of the experimental $\sigma_a^{exp}(\rho)$ and $\sigma_p^{exp}(\rho)$ values by the theoretical expressions given by the Equations (2) and (3). Due to the fact that to the $\sigma_p^{exp}(\rho)$ values contribute all three stress components, only these experimental data can be used in the regression analysis. Substituting the Equations (17), (18), and (19) into the Equations (2) and (3) the overestimated set of linear equations is obtained:

$$\begin{aligned} &t_0K_{t0}(r_i) + t_4K_{t4}(r_i) + r_4K_{r4}(r_i) + a_4K_{a4}(r_i) = \\ &= K(r_i)\sigma_p^{exp}(r_i) - K_R^{exp}(r_i), \quad i = 1,2...N_{exp} \end{aligned} \tag{20}$$

where N_{exp} is the number of experimental points $\sigma_p^{exp}(r_i)$ measured at coordinate r_i and the auxiliary K-functions are defined as follows:

$$K(r) = \sqrt{R^2 - r^2} \tag{21}$$

$$K_{t0}(r) = K(r) - \frac{r^2K(r)}{R^2} + \frac{K(r)^3}{R^2} \tag{22}$$

$$K_{t4}(r) = r^4K(r) + \frac{r^2K(r)^3}{3} - R^2r^2K(r) \tag{23}$$

$$K_{r4}(r) = \frac{r^2K(r)^3}{3} + \frac{K(r)^5}{5} - \frac{R^2K(r)^3}{3} \tag{24}$$

$$K_R^{exp}(r) = \sigma_p^{exp}(R)\left[\frac{2r^2K(r)}{R^2} + \frac{K(r)^3}{3R^2} - K(r)\right] \tag{25}$$

Using the standard methods of multilinear regression analysis the best estimates of the unknown values of t_0, t_4, r_4, and a_4 are obtained together with the corresponding standard deviations and covariance matrix. These statistical parameters enable the calculation of confidence intervals for each estimated stress profile.

EXPERIMENTAL

Measuring equipment consisted of polarizer, monochromatic filter, quarter wave plate, and analyzer connected to the microscope (NIKON Eclipse ME600). CCD camera was used to record images after compensation. The Lucia® image analysis software (LIM, Ltd. Prague) was used. The cylindrical samples with the diameter of 7 mm and height of 11 mm were obtained from stems of the industrially produced stemware. The perpendicular measurement was made in immersion liquid in the central sample cross section. The photo-elastic constant of the barium crystal glass used (produced by RONA glassworks, Lednické Rovne, Slovakia) was calculated using the method of Demkina [5]. The value of B = 3.0·10-12 Pa-1 was obtained.

Measurement in axial direction

The obtained experimental values of $\sigma_a^{exp}(\rho)$ are plotted in Figure 2. The values obtained by measurement in axial direction were not used for obtaining the particular stress profiles. The only exception is the $\sigma_a^{exp}(R)$ value that was used to estimate the t_2 value (see the Equation (11)). On the other hand these values were used for independent check of the quality of stress profiles obtained by regression analysis of the perpendicular measurement experimental data. Moreover it was statistically proved that the radial dependence of $\sigma_a^{exp}(\rho)$ is quadratic, i.e.

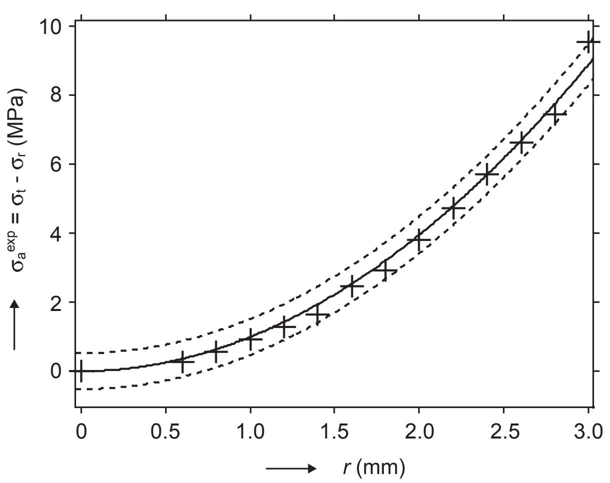

Figure 2. Experimental values obtained by the measurement in axial direction $\sigma_a^{exp}(\rho)$ and their quadratic approximation.

$$\sigma_a^{exp}(\rho) = q_2\rho^2 = (t_2 - r_2)\rho^2 \qquad (26)$$

where q_2 = 0.990 ± 0.031 (Figure 2). Thus the difference between the fourth powers terms from the Equations (5) and (6) is negligible.

Measurement in perpendicular direction

The measured values of $\sigma_p^{exp}(r)$ are summarized in Table 1 together with their regression fit.

Table 1. Measured and fitted values of σ_p (r). Δ is corresponding absolute deviation.

r (mm)	$\sigma_p^{exp}(r)$ (MPa)	$\sigma_p^{regression}(r)$ (MPa)	Δ
0.6	3.48	3,47	0,014
0.8	3.08	3,05	0,026
1	2.68	2,53	0,154
1.2	1.67	1,88	0,208
1.4	1.25	1,12	0,135
1.6	0	0,23	0,233
1.8	-0.67	-0,77	0,095
2	-1.91	-1,88	0,028
2.2	-2.94	-3,12	0,175
2.4	-4.51	-4,47	0,043
2.6	-6.19	-5,94	0,254
2.8	-7.38	-7,52	0,142
3	-9.42	-9,23	0,193

RESULTS AND DISCUSSION

The multilinear regression analysis of experimental $\sigma_p^{exp}(r)$ according to the set of linear Equation (20) resulted in statistically non-significant values of the fourth order parameters t_4, r_4, and a_4. Similarly all attempts to retain at least one of fourth order parameters in the regression analysis were not successful. This result is in harmony with the finding of quadratic character of the $\sigma_a^{exp}(\rho)$ dependence presented in Figure 2. Finally, only one unknown parameter, i.e. t_0, was estimated by the regression analysis. This procedure resulted in the value of t_0 = (8.009 ± 0.176) MPa. The resulting fit is, together with the 95 % confidence limit, illustrated in Figure 3. The residual sum of squares of 2.363 MPa²·mm² corresponds to the standard deviation of approximation of sapr = 0.44 MPamm. The coefficients of Taylor series of individual stress components calculated for the regression estimate of t_0 are summarized in Table 2.

The quality of the obtained result may be seen in Figure 4 where the experimental results of axial measurements (not used in the regression analysis) are compared with the theoretical curve calculated from the data of Table 2 according the Equations (5) and (6). The

Table 2. Taylor coefficient characteristics.

Parameters	
t_0 = 8.009 ± 0.176	(MPa)
t_2 = -1.797 ± 0.014	(MPa/mm^2)
t_4 = 0	(MPa/mm^4)
r_0 = 8.009 ± 0,176	(MPa)
r_2 = -0.654 ± 0.014	(MPa/mm^2)
r_4 = 0	(MPa/mm^4)
a_0 = 14	(MPa)
a_2 = -2.285	(MPa/mm^2)
a_4 = 0	(MPa/mm^4)

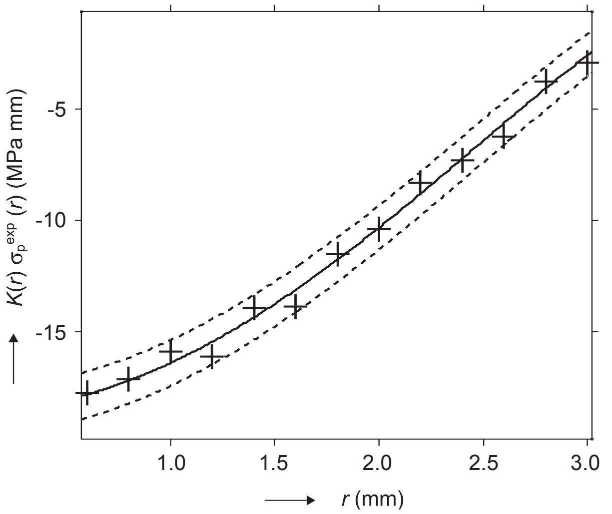

Figure 3. Obtained fit of the Equation (20). Dotted lines correspond to 95 % confidence limits.

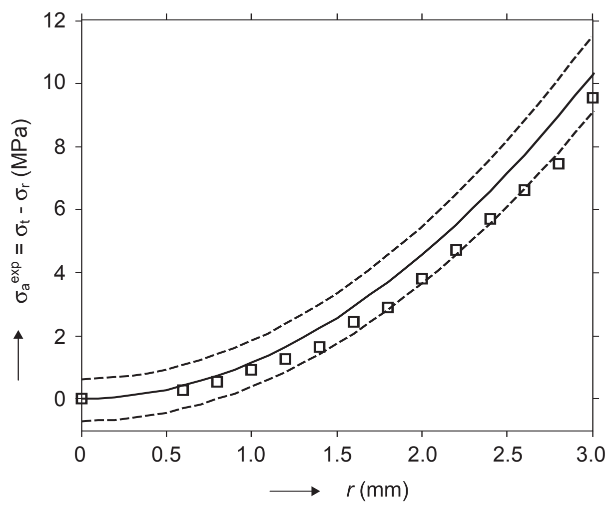

Figure 4. Comparison of measured (symbols) and calculated (thick line) stress values related to measurement in axial mode. Dotted lines correspond to 95 % confidence limits.

obtained fit may be considered as very promising and thus the developed method can be used for the study of stress relaxation processes in cylindrical samples.

The courses of individual profiles of tangential, radial and axial stress obtained using the data from Table 2 are plotted in Figure 5.

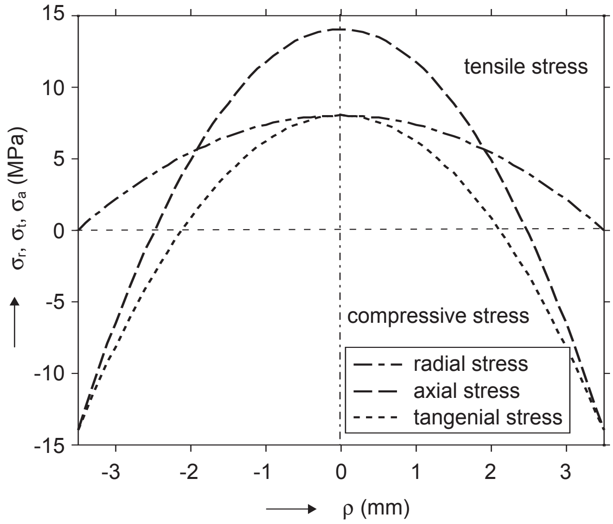

Figure 5. Separated stress component profiles obtained from measurement in the direction perpendicular to the rotational axis

CONCLUSION

Procedure of individual stress component separation was implemented on measured stress by Senarmont method on a cylindrical glass sample. Calculated radial and tangential stress profiles were successfully verified with the help of measurement in axial axis. The achieved value of deviations between calculated and experimental results are caused by non-uniform cooling of the investigated glass probe. In spite of the fact that axial symmetry of the probe was disturbed under those circumstances, the accepted approach relying on symmetrical Taylor function proved to be robust enough to ensure reliable results.

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MERANIE PRIEBEHU NAPÄTIA
V OSOVOSYMETRICKEJ SKLENENEJ VZORKE

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Na určenie priebehov tangenciálnej, radiálnej a axiálnej zložky napätia v sklenenej vzorke bola použitá Sénarmontova metóda. Bola meraná závislosť fázového rozdielu na radiálnej súradnici pri šírenie svetla vzorkou v axiálnom a kolmom smere, s rešpektovaním osovej symetrie vzorky. Bola použitá numerická analýza založená na Taylorovom rozvoji jednotlivých zložiek napätí v radiálnom smere v kombinácií s regresnou analýzou pre hodnotenie zložiek napätia. Štatistické spracovanie experimentálnych dát ukázalo, že pri cylindrickej symetriu je pre opis rozloženia napätia postačujúci Taylorov rozvoj do kvadratického člena. 95% interval spoľahlivosti získaných priebehov napätí je dostatočne úzky a súhlasí s relatívnou chybou jednotlivých zložiek napätia v intervale 5-10%.